Distributed Computing on the (Fruit) Fly

Yuval Emek

Technion - Israel Institute of Technology

The 1st Workshop on Biological Distributed Algorithms
Jerusalem, October 2013

Synopsis

Distributed network algorithms

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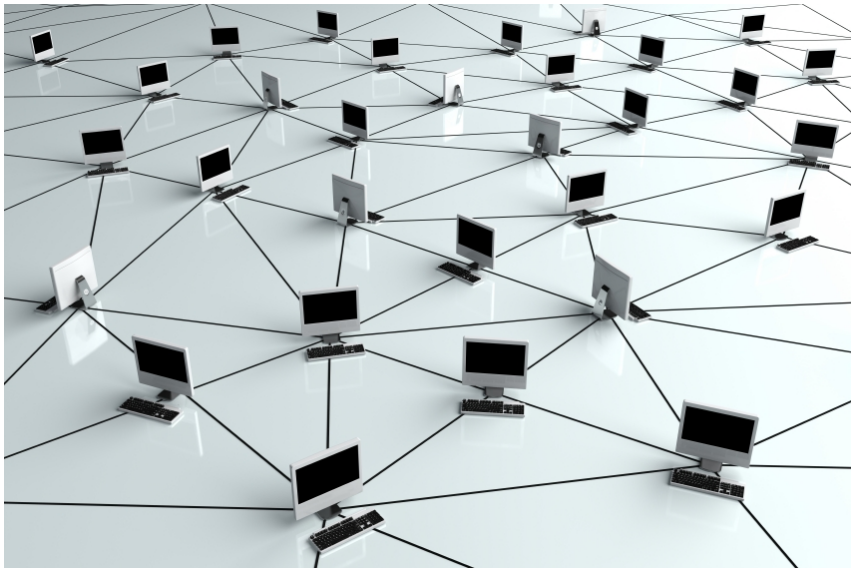

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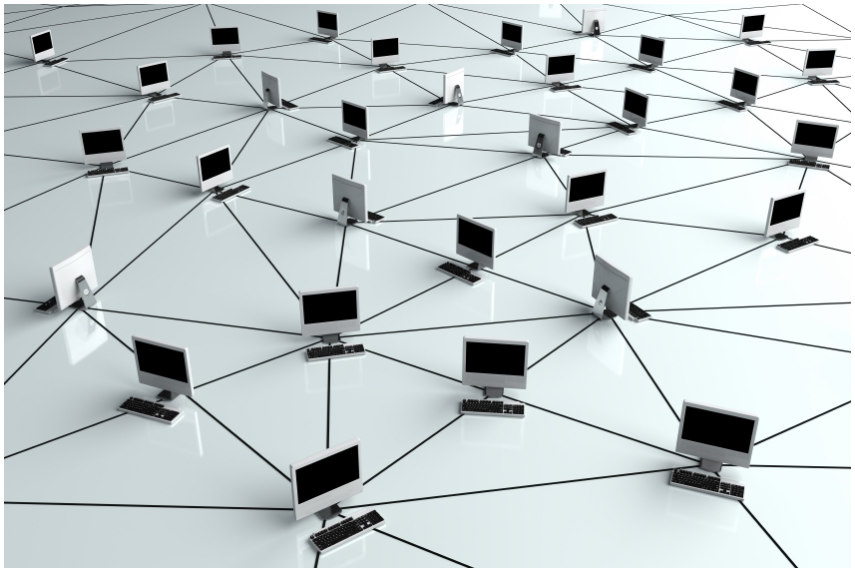
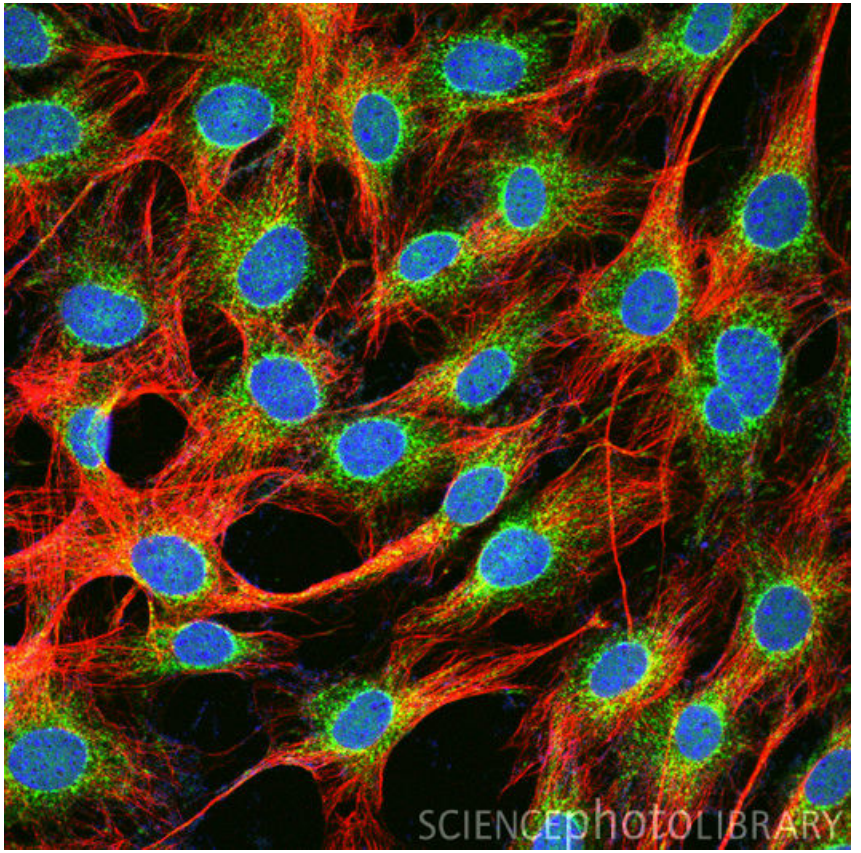
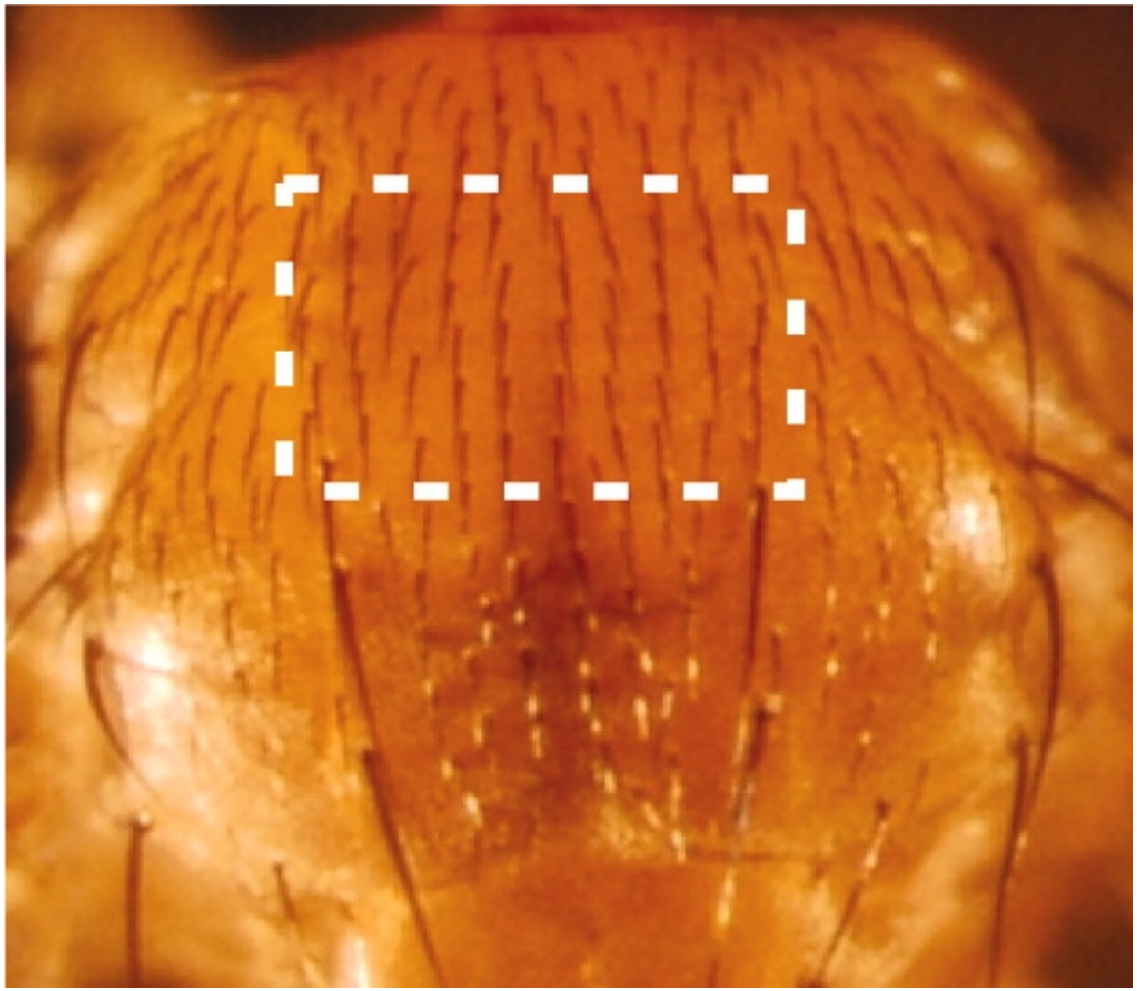




Mission: theory of distributed computing in biological cellular networks

Motivation

Selection of sensory organ precurser (SOP) cells = solving MIS
[Afek, Alon, Barad, Hornstein, Barkai, Bar-Joseph 11]



1. Cells as computing devices
2. Abstract distributed computing models
3. Networked finite state machines
4. Results
   - MIS algorithm
5. Conclusions

Local computation

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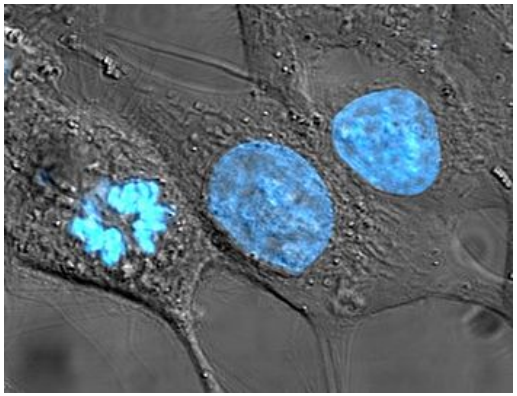
Nucleus: analogous to central processing unit



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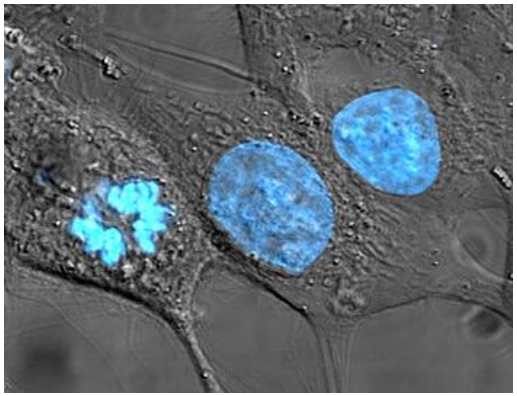

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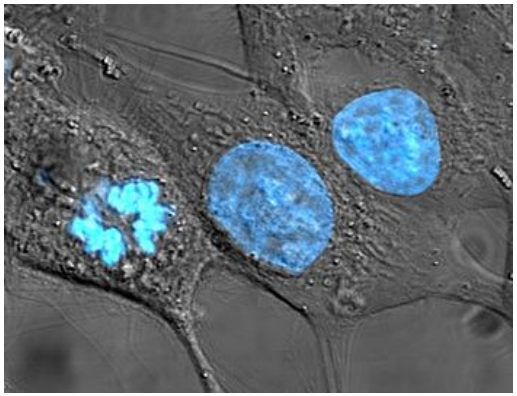

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- Instructions = genes (DNA substrings)

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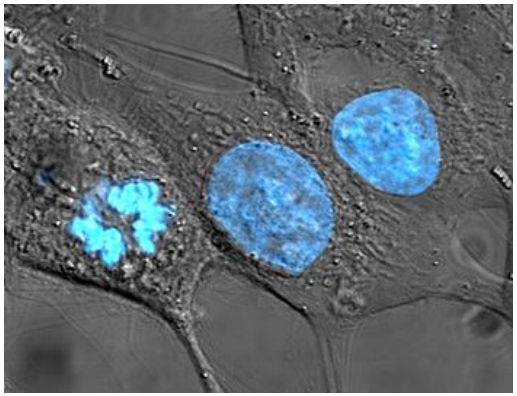

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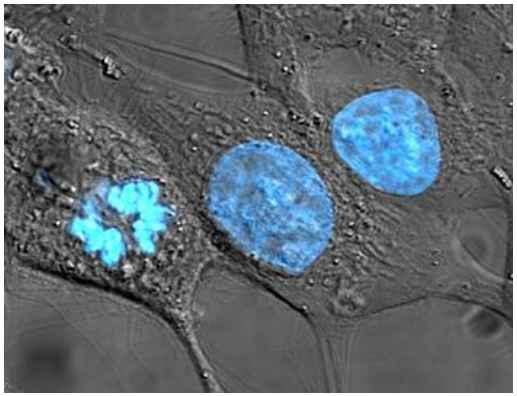

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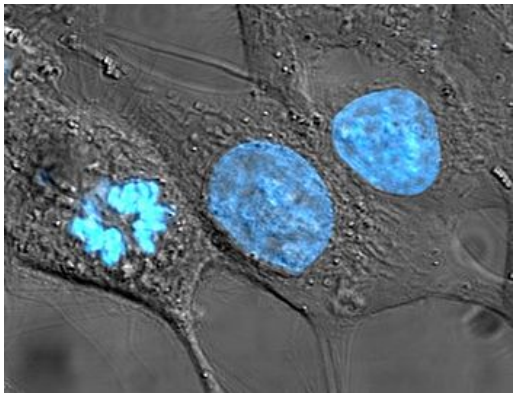

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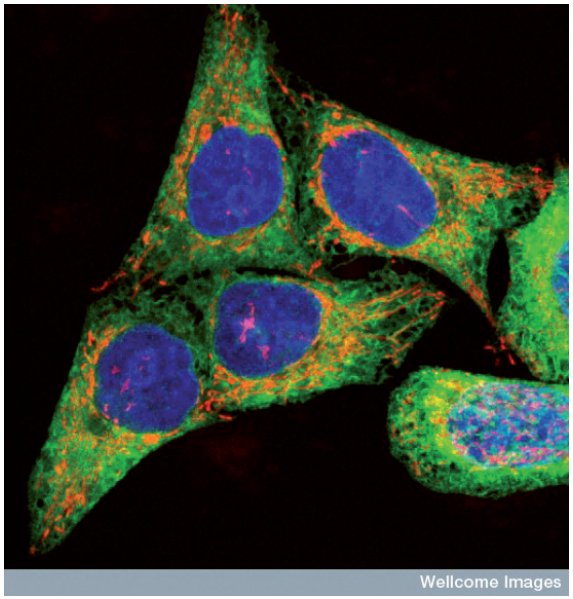

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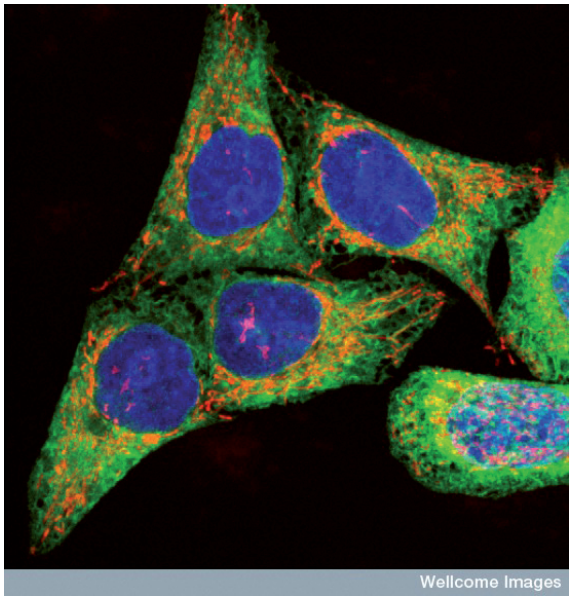

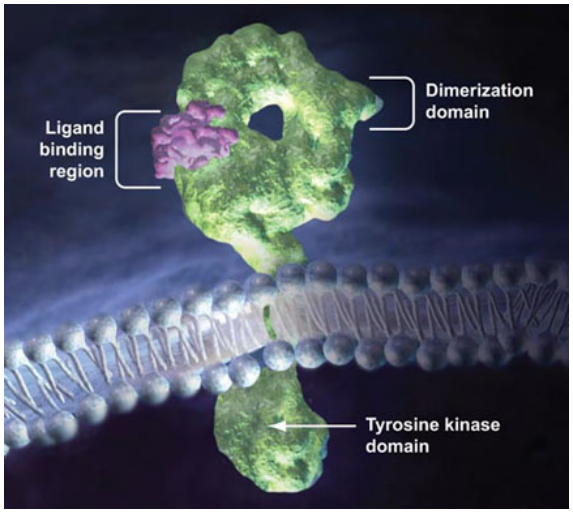

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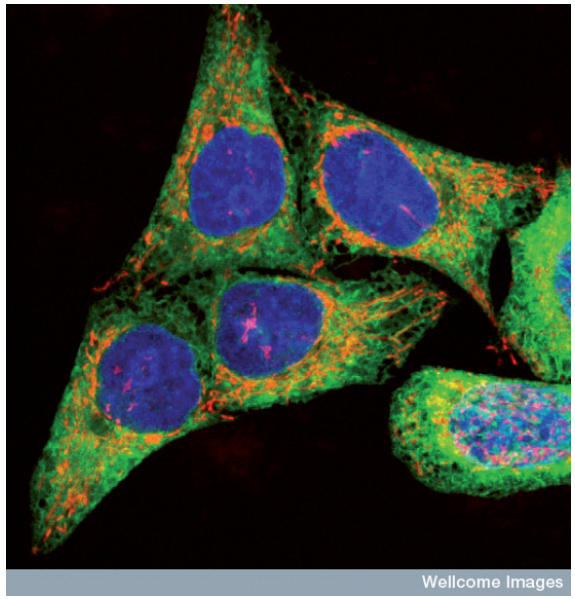

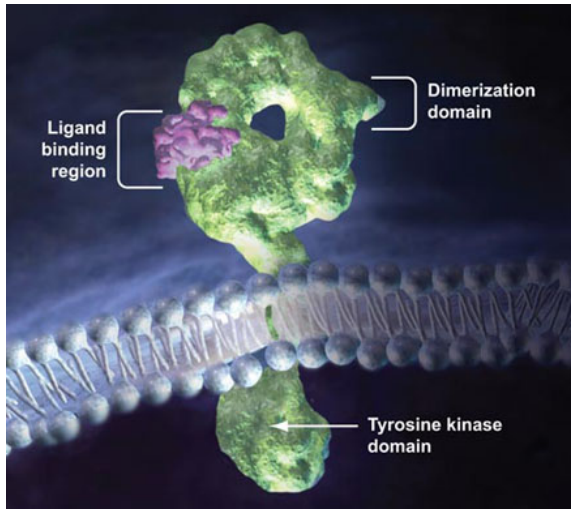

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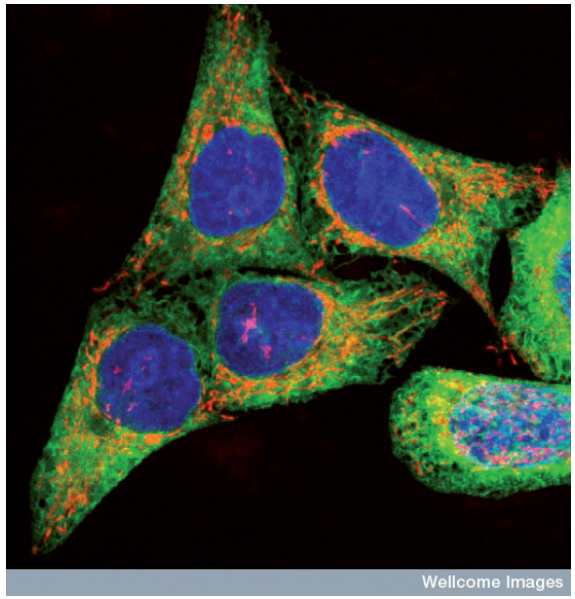

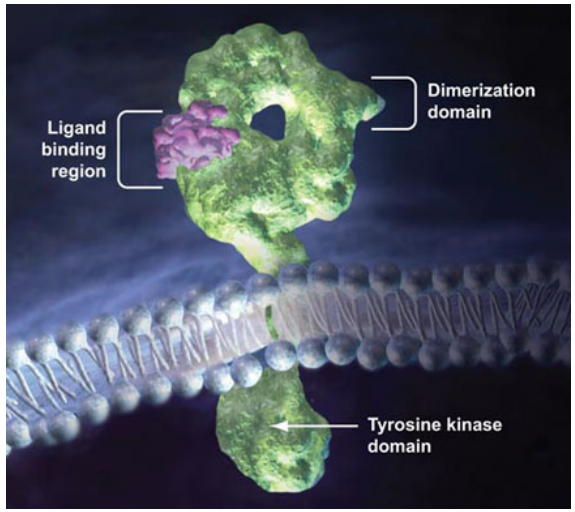

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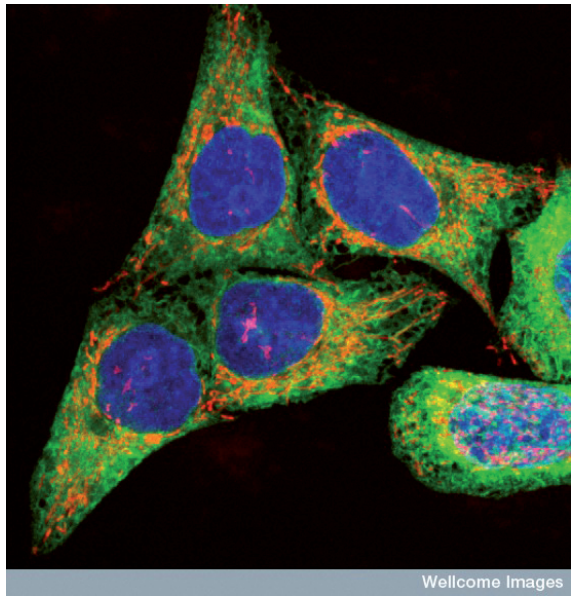

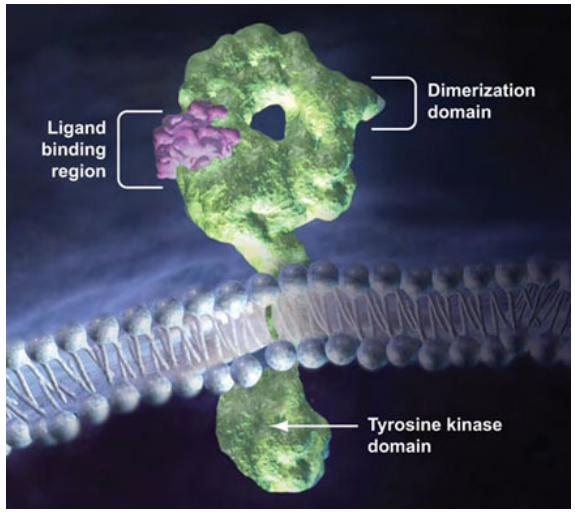

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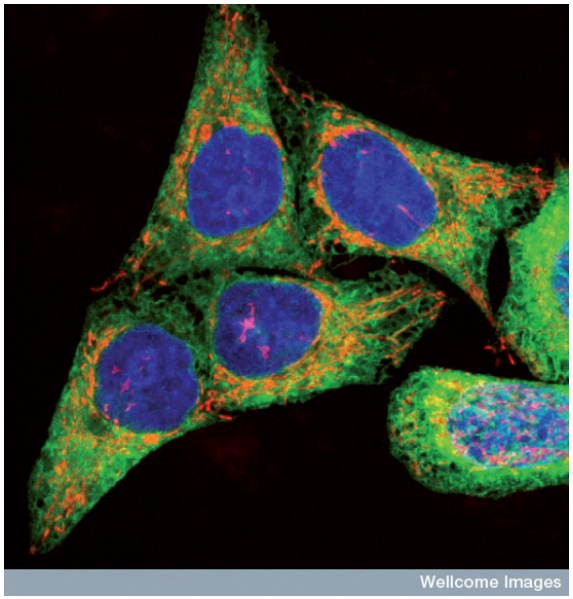

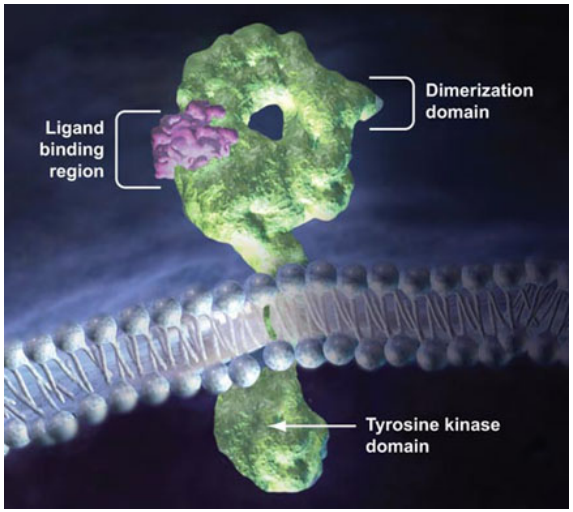

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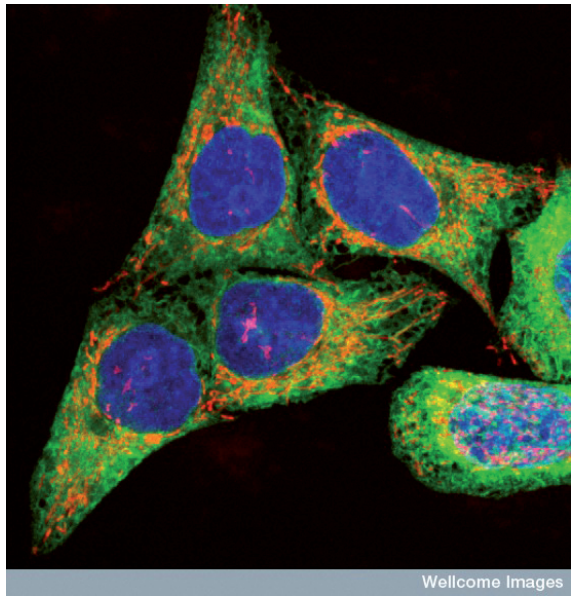
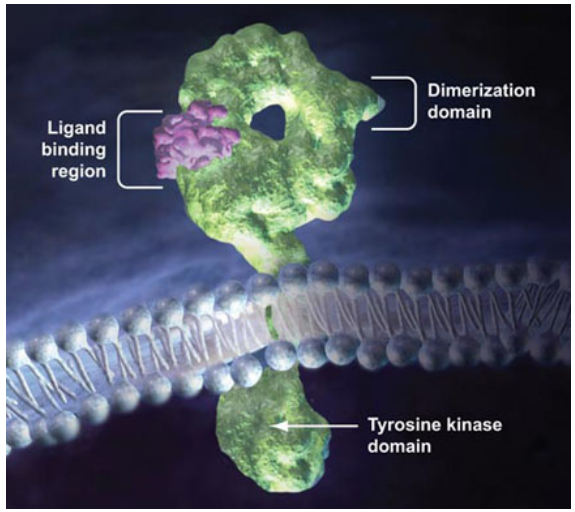
- Gap junction/receptor = port
- No sense of direction
  - all neighbors look the same





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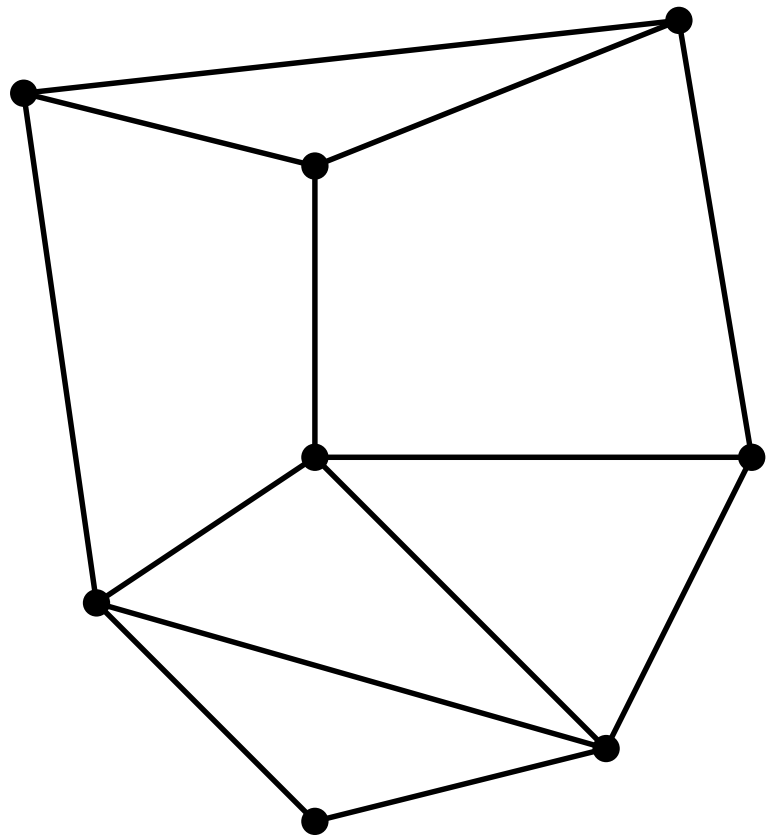

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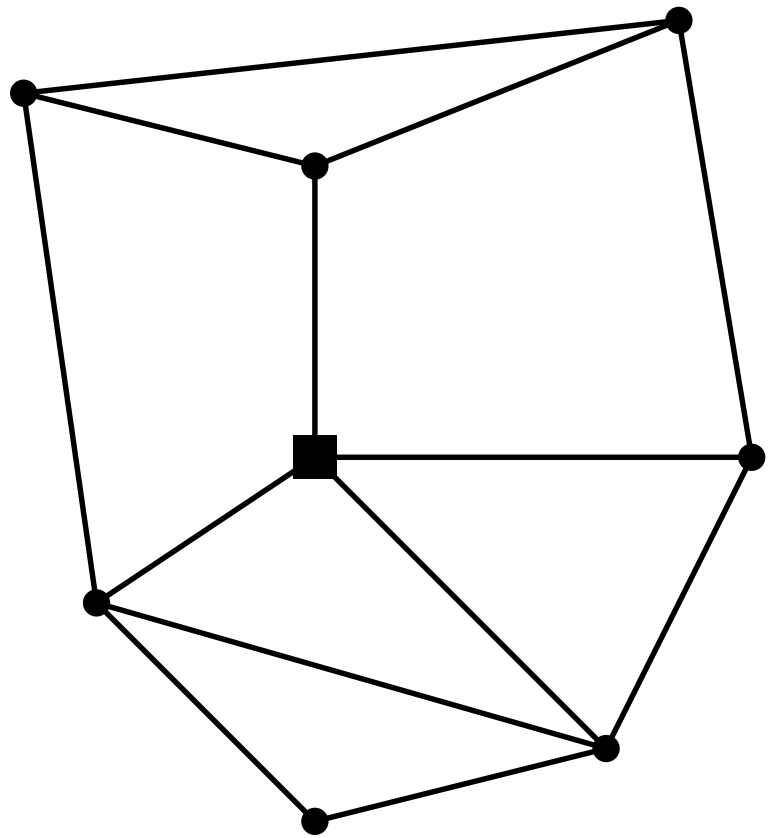

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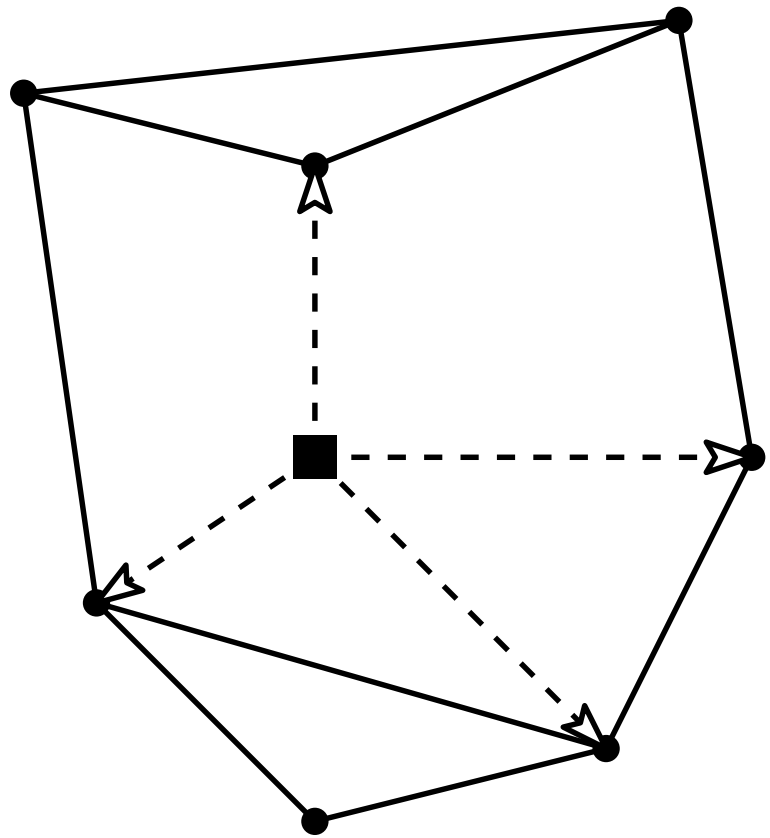

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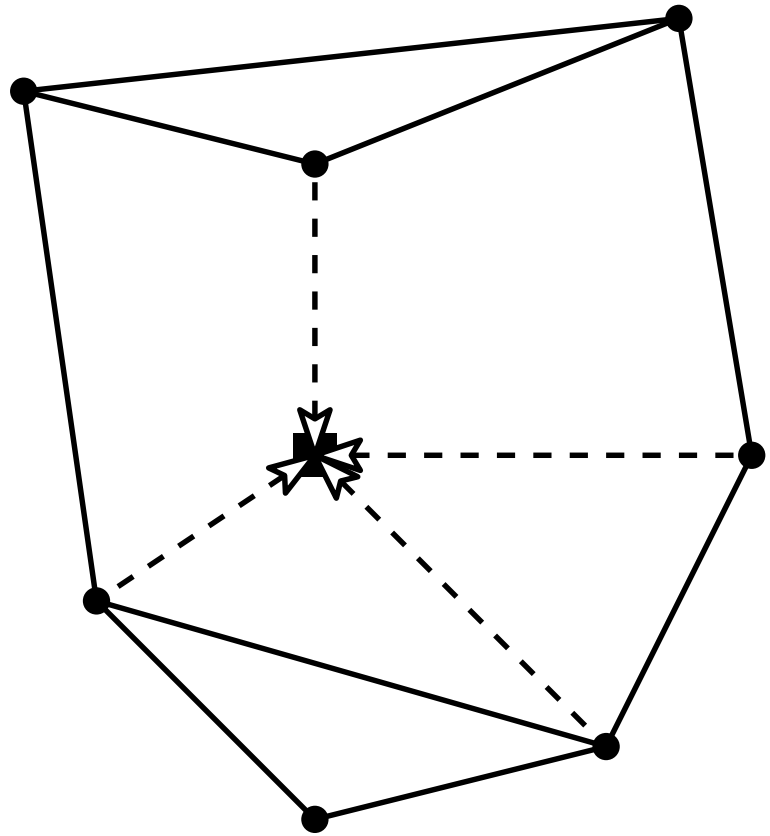

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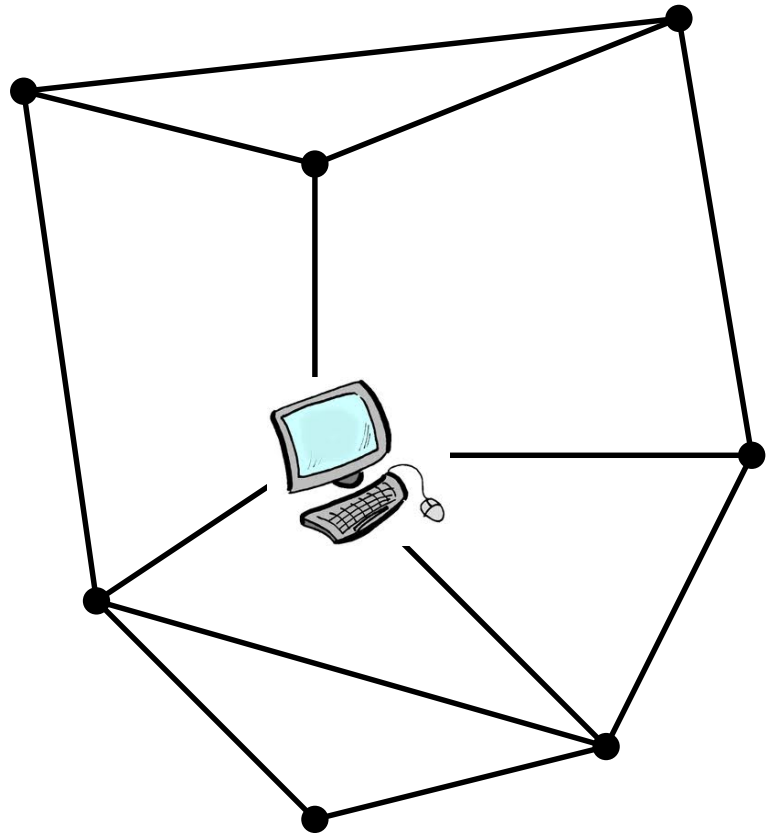

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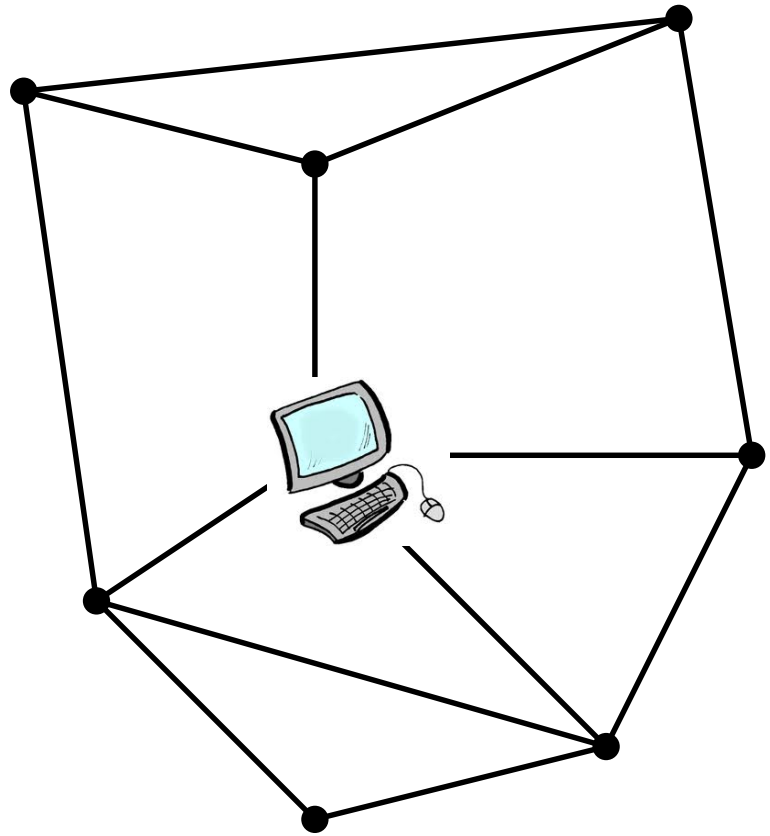

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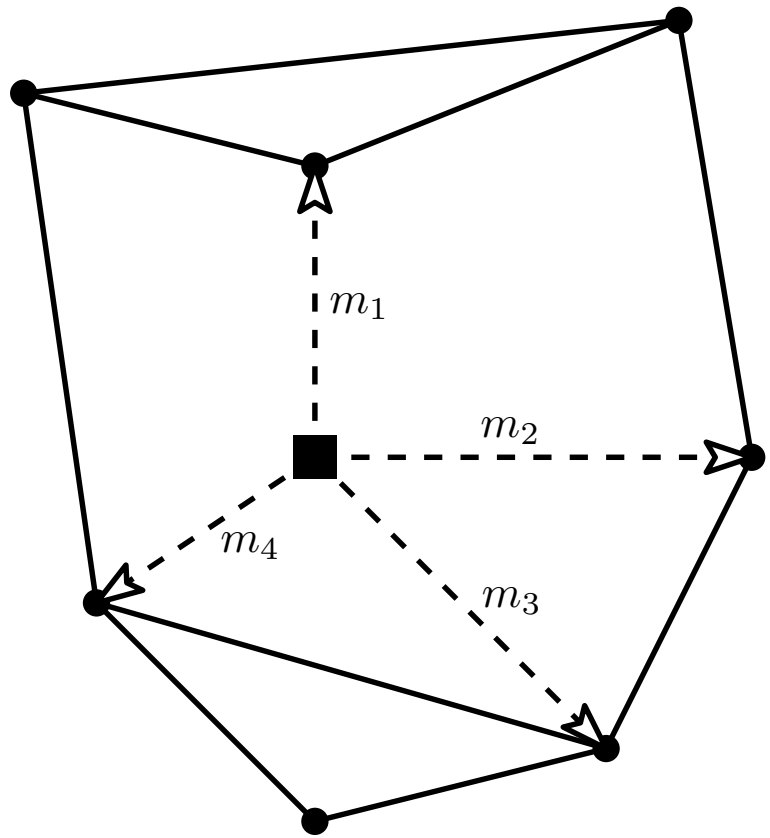

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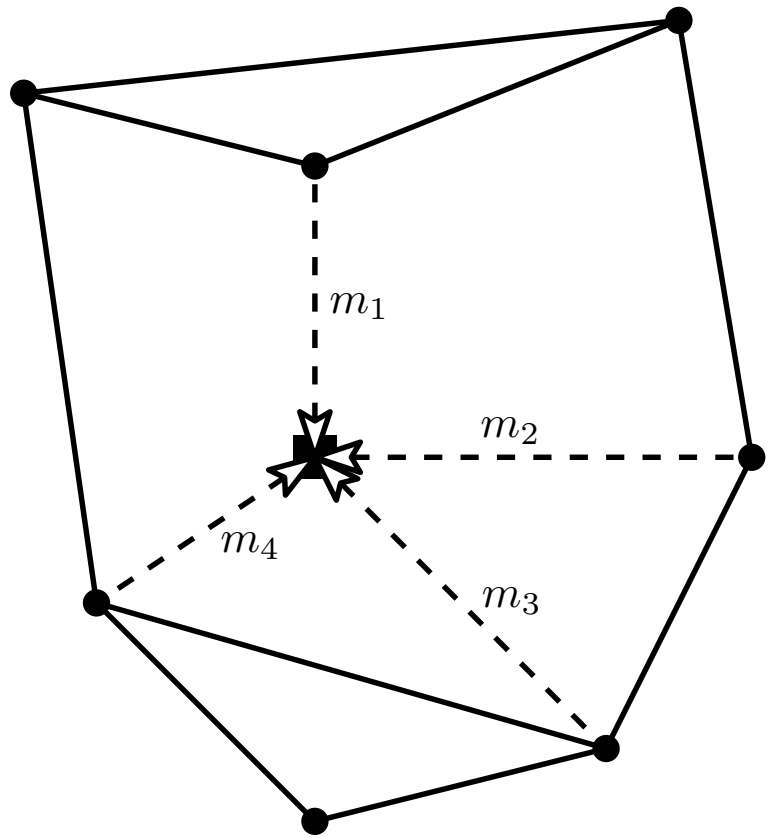

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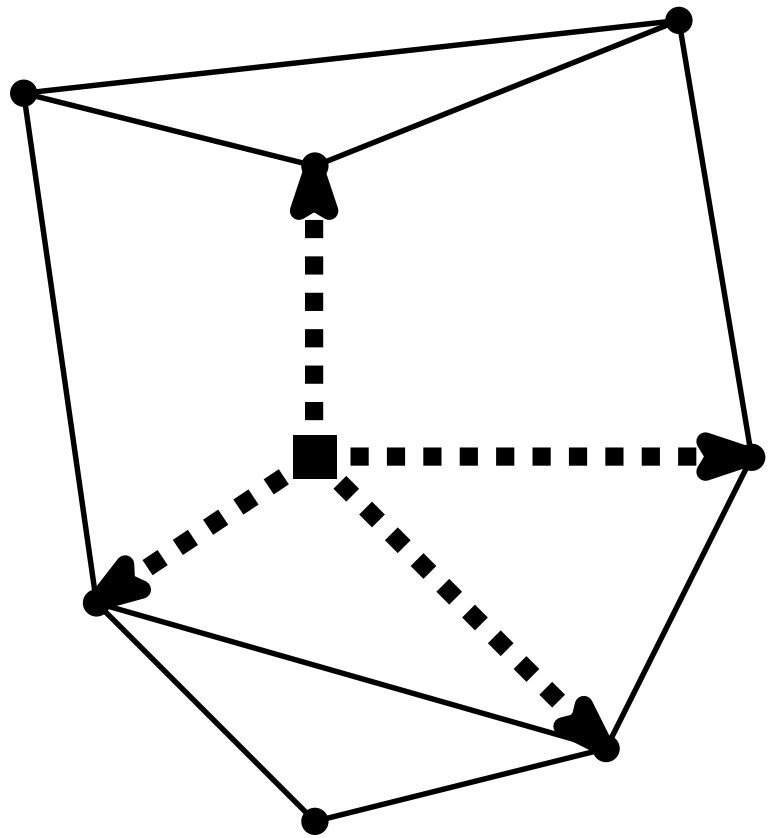

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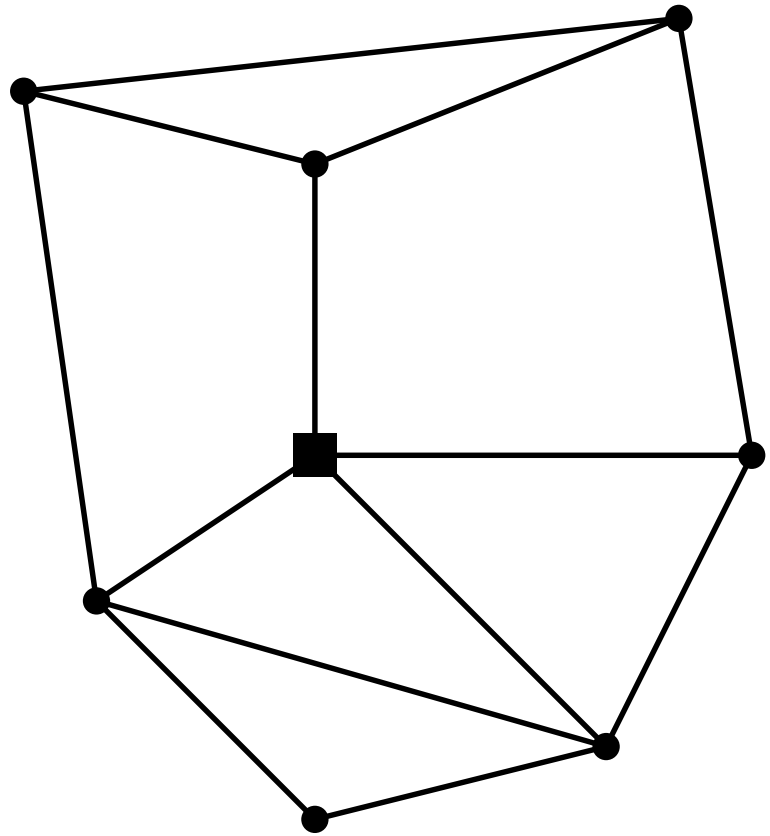
The beeping model

Introduced in [Cornejo, Kuhn 10]



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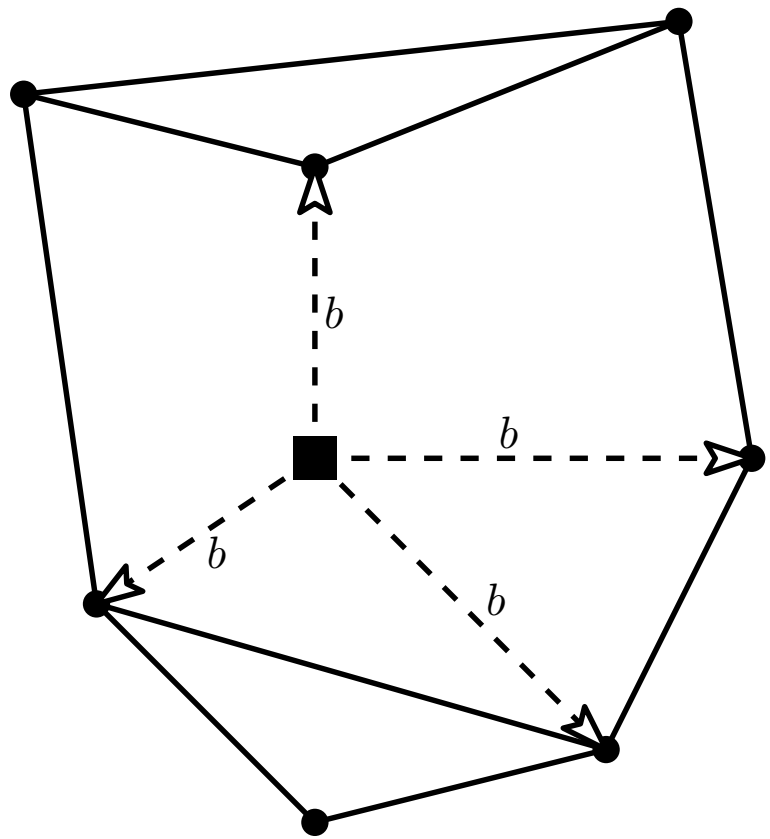

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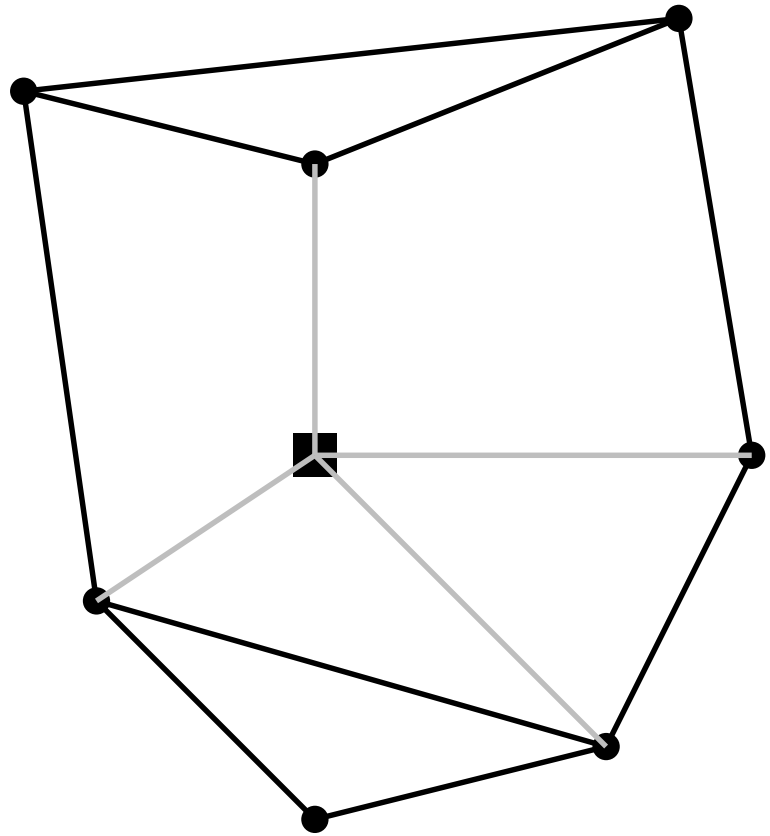

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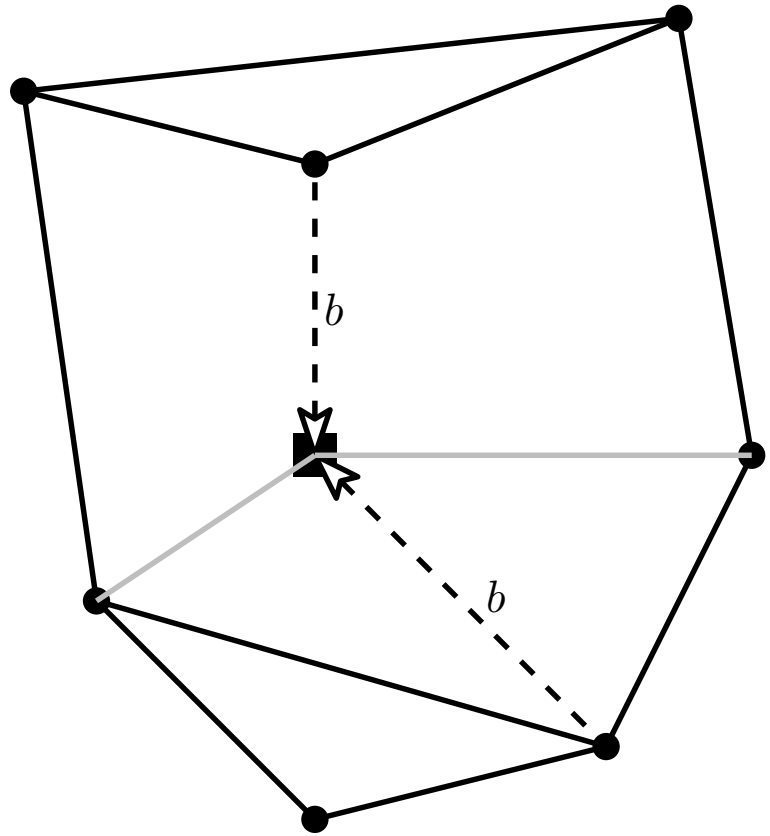

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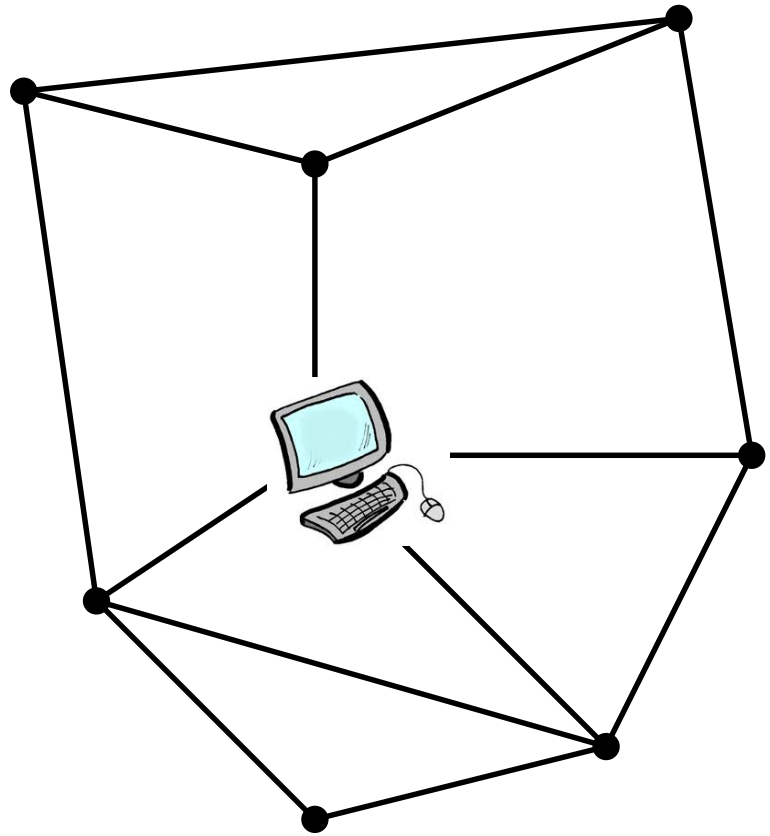

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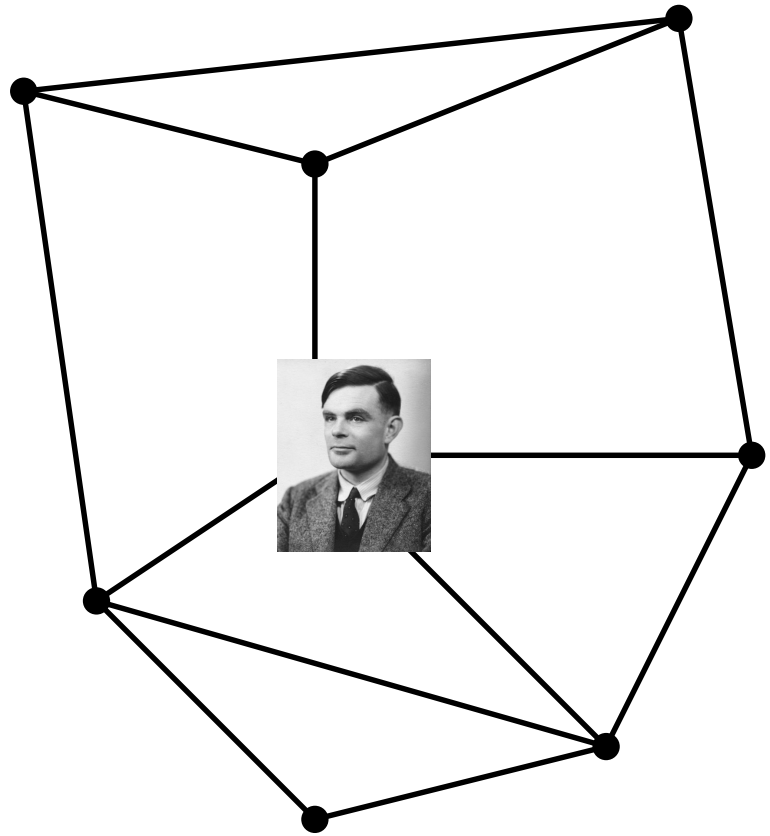

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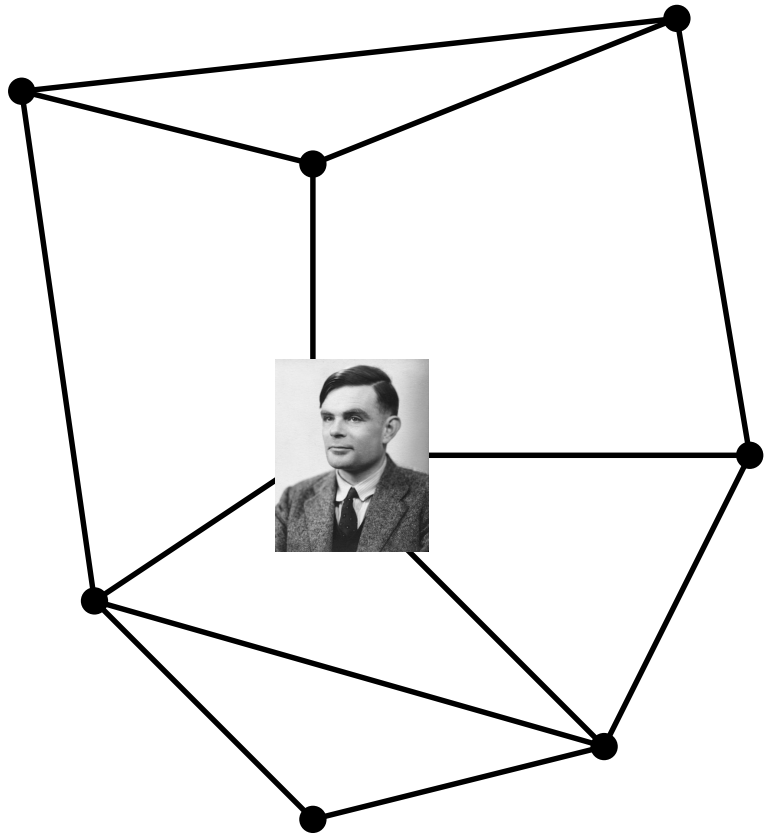

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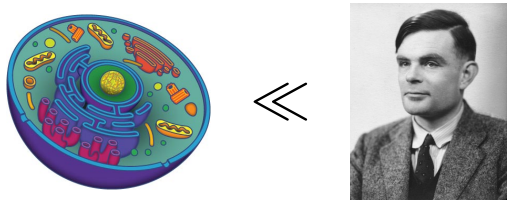

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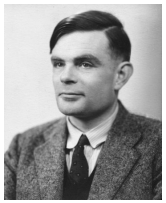

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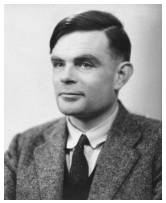

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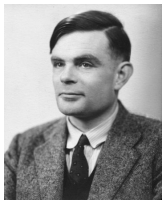

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[Benenson, Paz-Elizur, Adar, Keinan, Livneh, Shapiro 01]

Perhaps we should aim for a network of FSMs?

Cellular automata

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Infinite grid of FSMs

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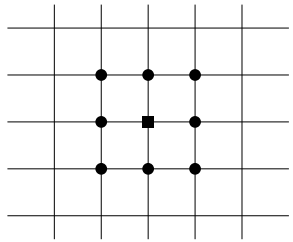
$q_{x,y}(t+1) \longleftarrow q_{x,y}(t), \{q_{x',y'}(t) : \text{grid neighbors } (x', y')\}$



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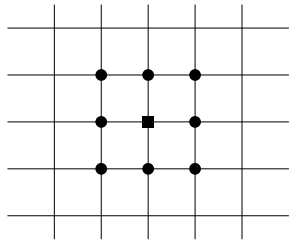

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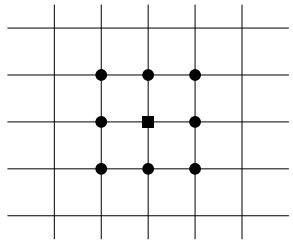

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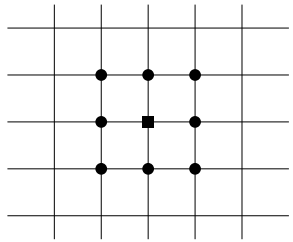

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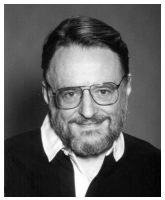

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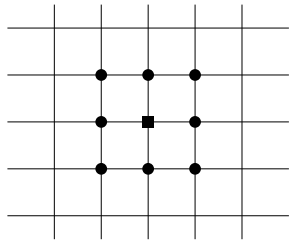

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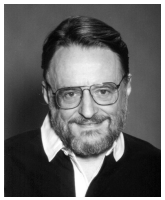
Game of life 

Digital physics  

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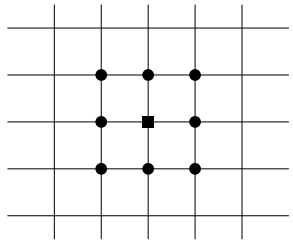

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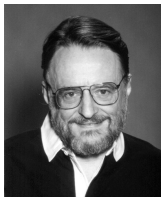

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 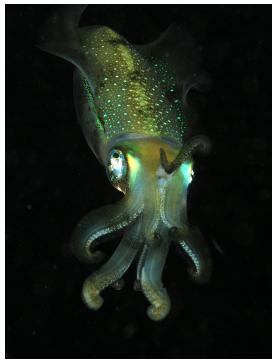

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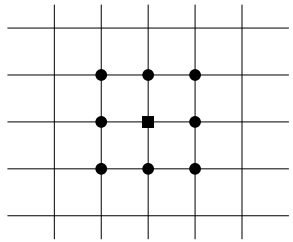

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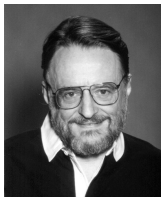

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 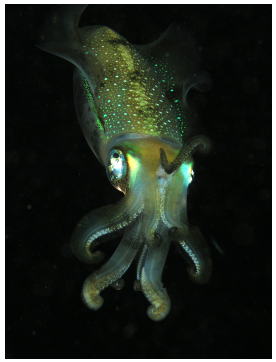

Highly regular topology

1. Cells as computing devices
2. Abstract distributed computing models
3. Networked finite state machines
4. Results
   - MIS algorithm
5. Conclusions

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- Maximal Matching in arbitrary graphs (small model modification)
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Additional general results

Theorem (Synchronizer)

Every nFSM algorithm designed to operate in a synchronous environment can be simulated in an asynchronous environment with a constant multiplicative run-time overhead.

Theorem (Computability)

In terms of their computational power, nFSM algorithms are (almost) equivalent to randomized linear-space Turing machines.

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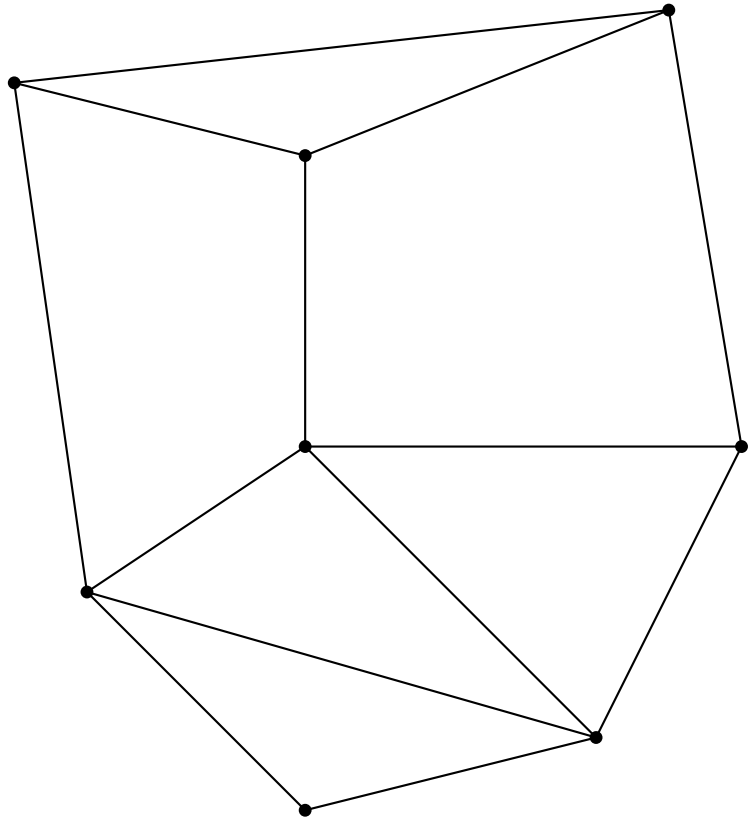

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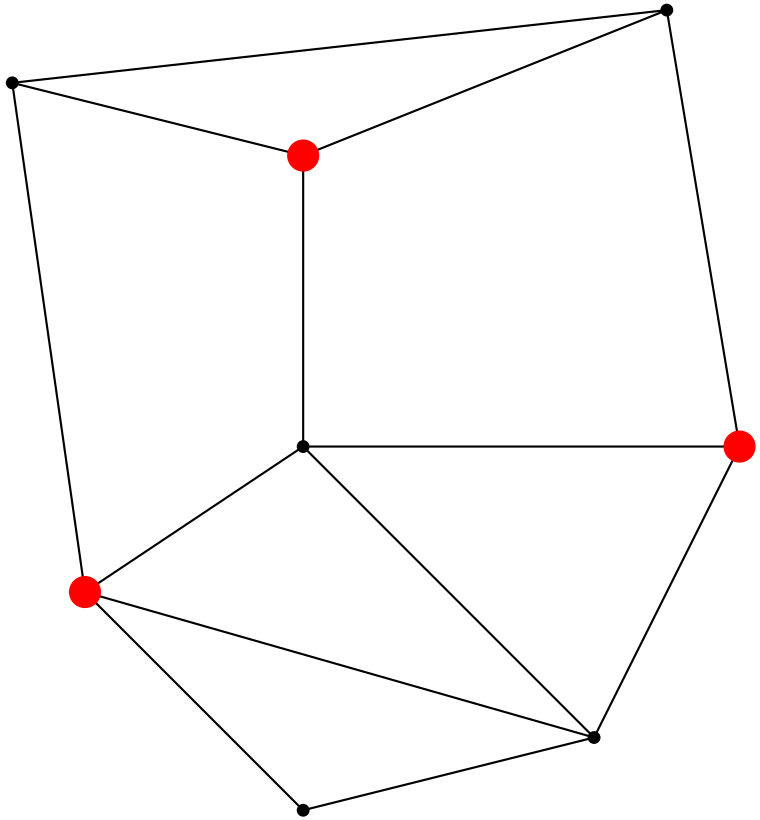

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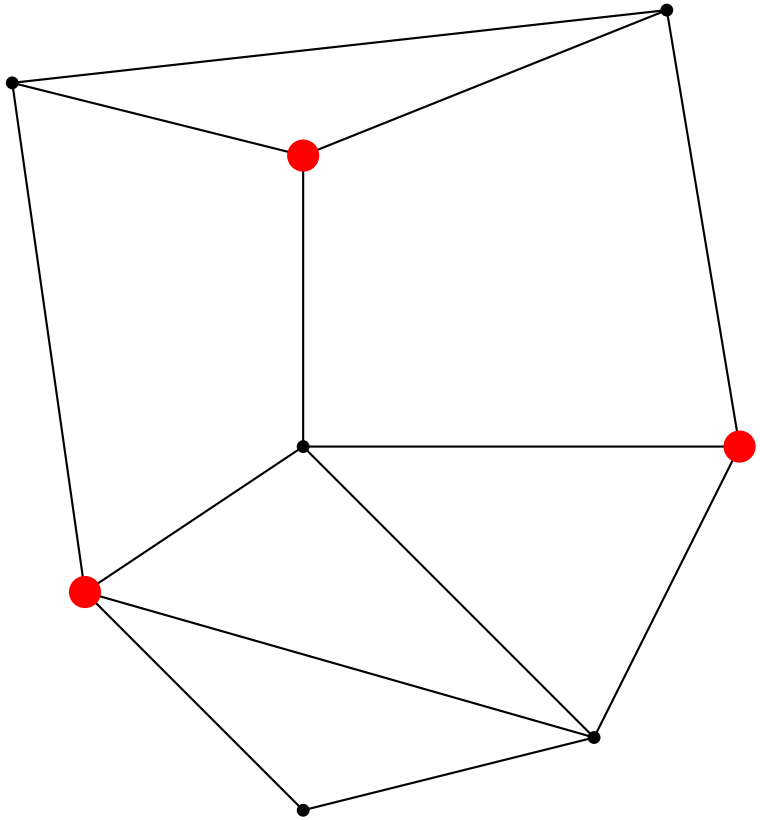

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input: (arbitrary) network
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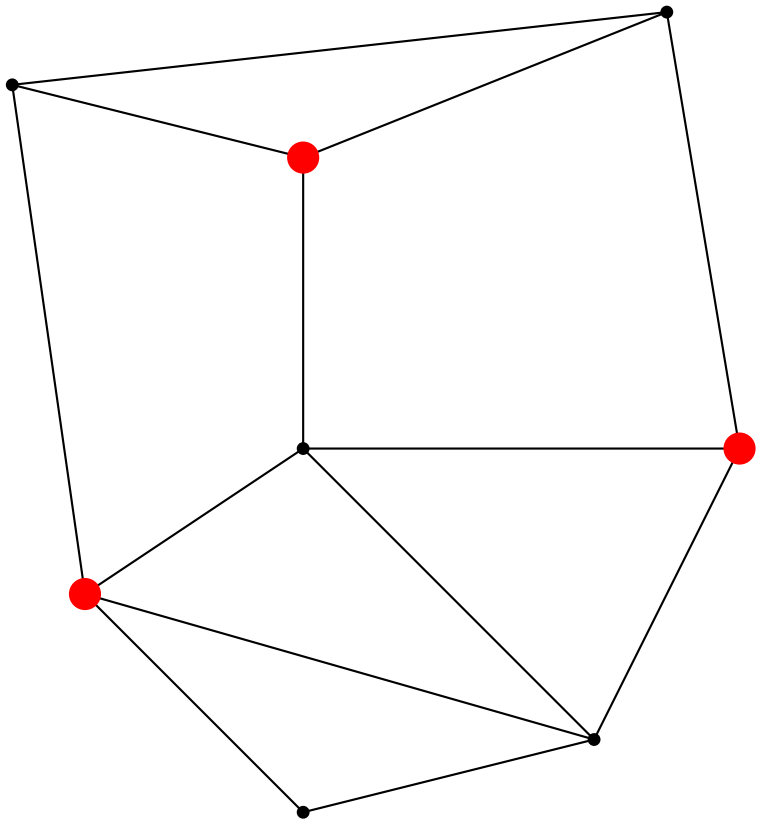

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- How can we decide if u joins MIS without long aligned phases?

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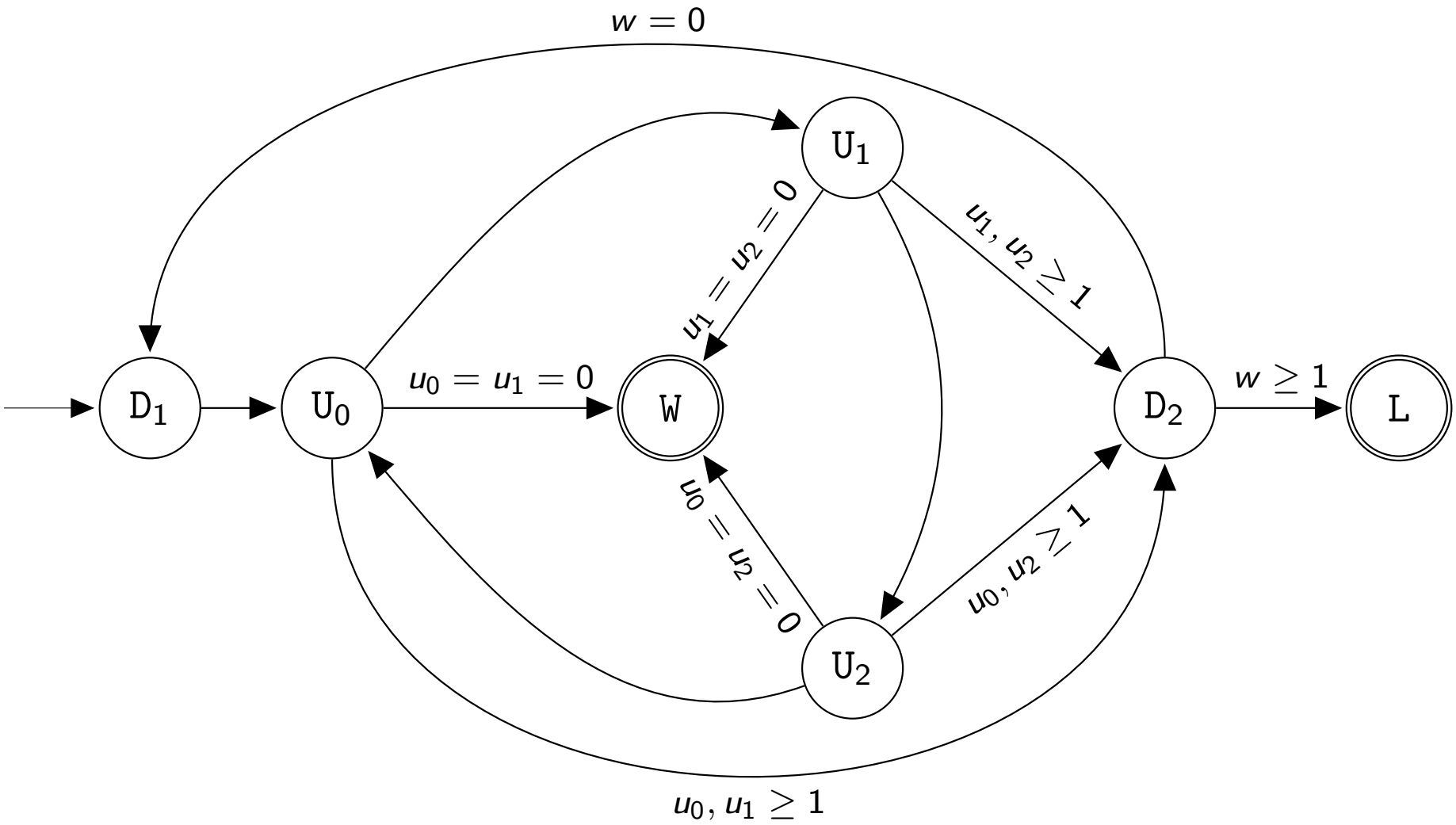

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תודה רבה