Distributed Computing on the (Fruit) Fly

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Distributed Computing Group
ETH Zurich

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Advances in Distributed Graph Algorithms (ADGA)
Salvador Bahia, Brazil

Synopsis

Distributed computing = power & limitations of computation in networks

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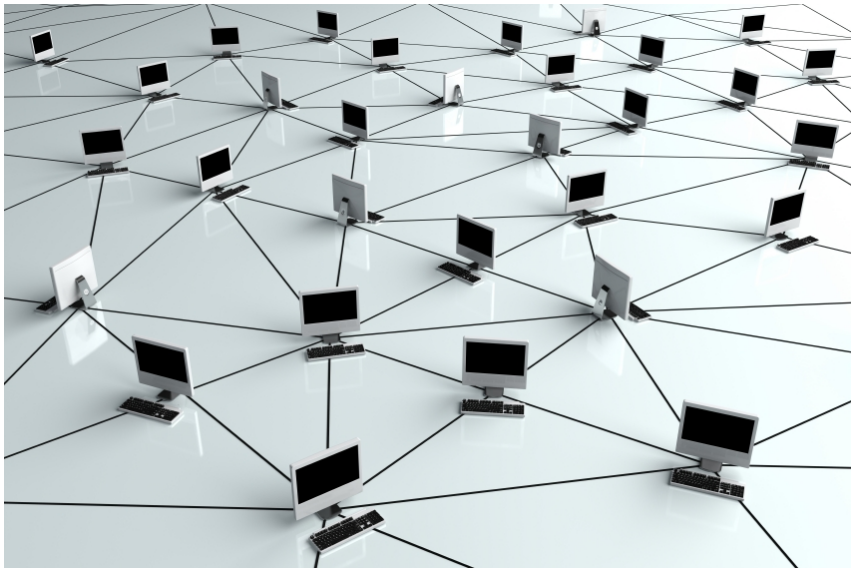

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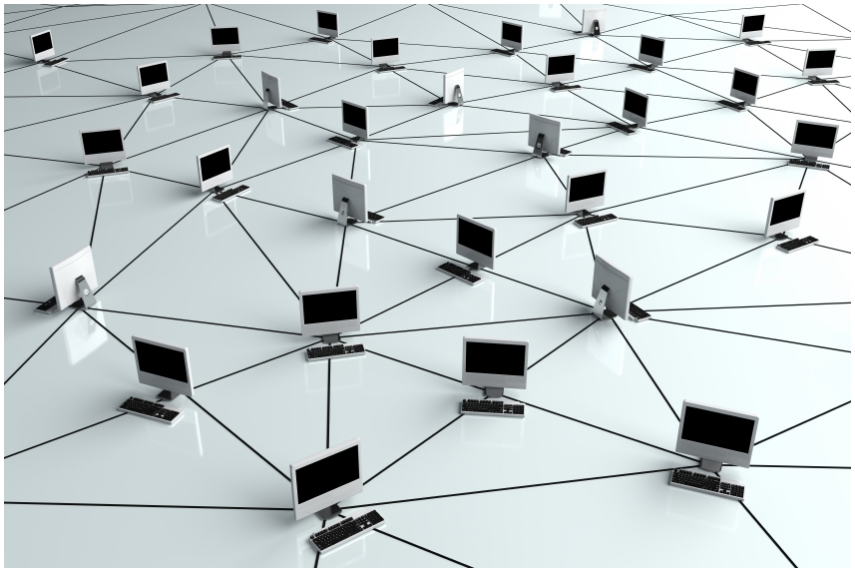

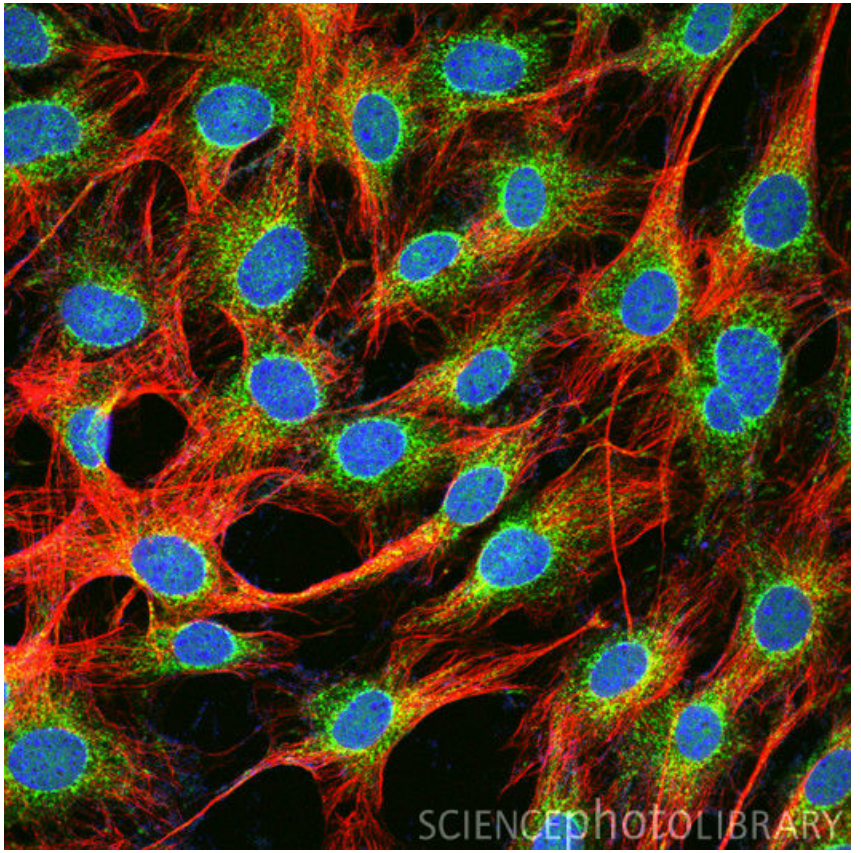

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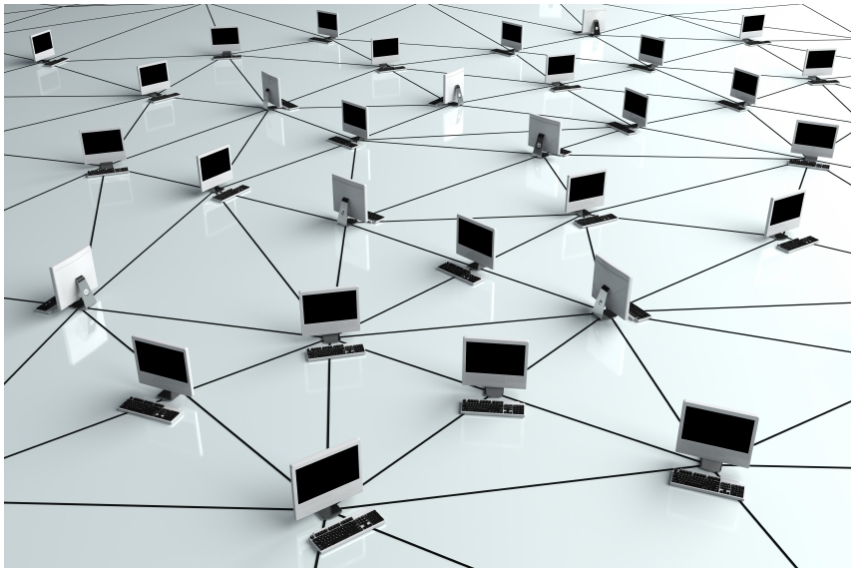
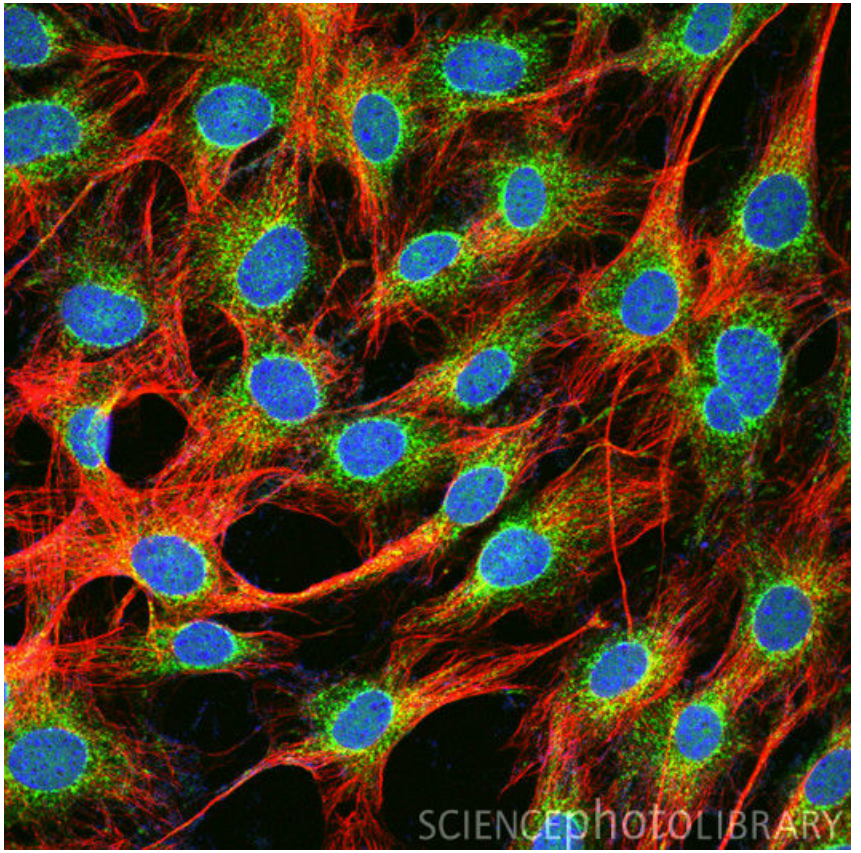
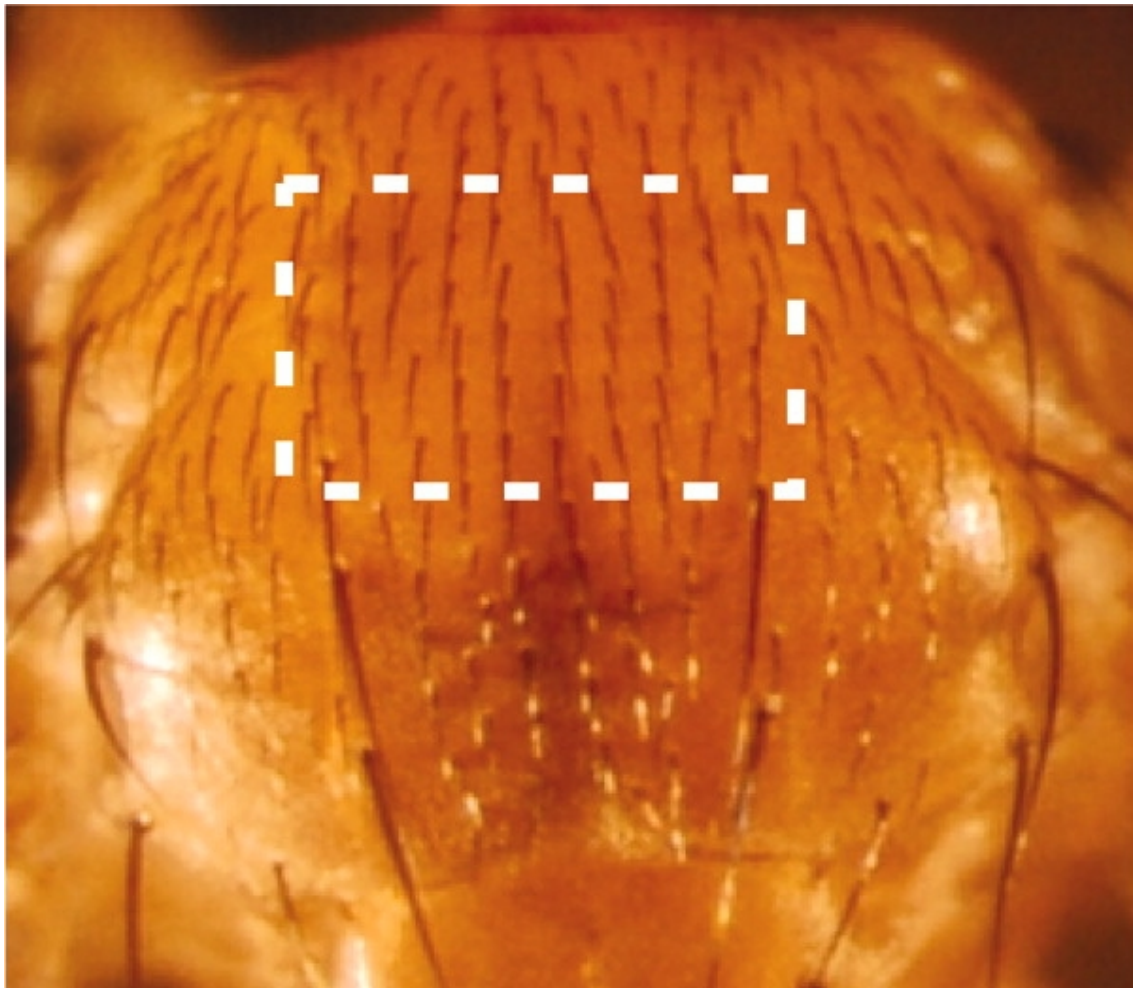




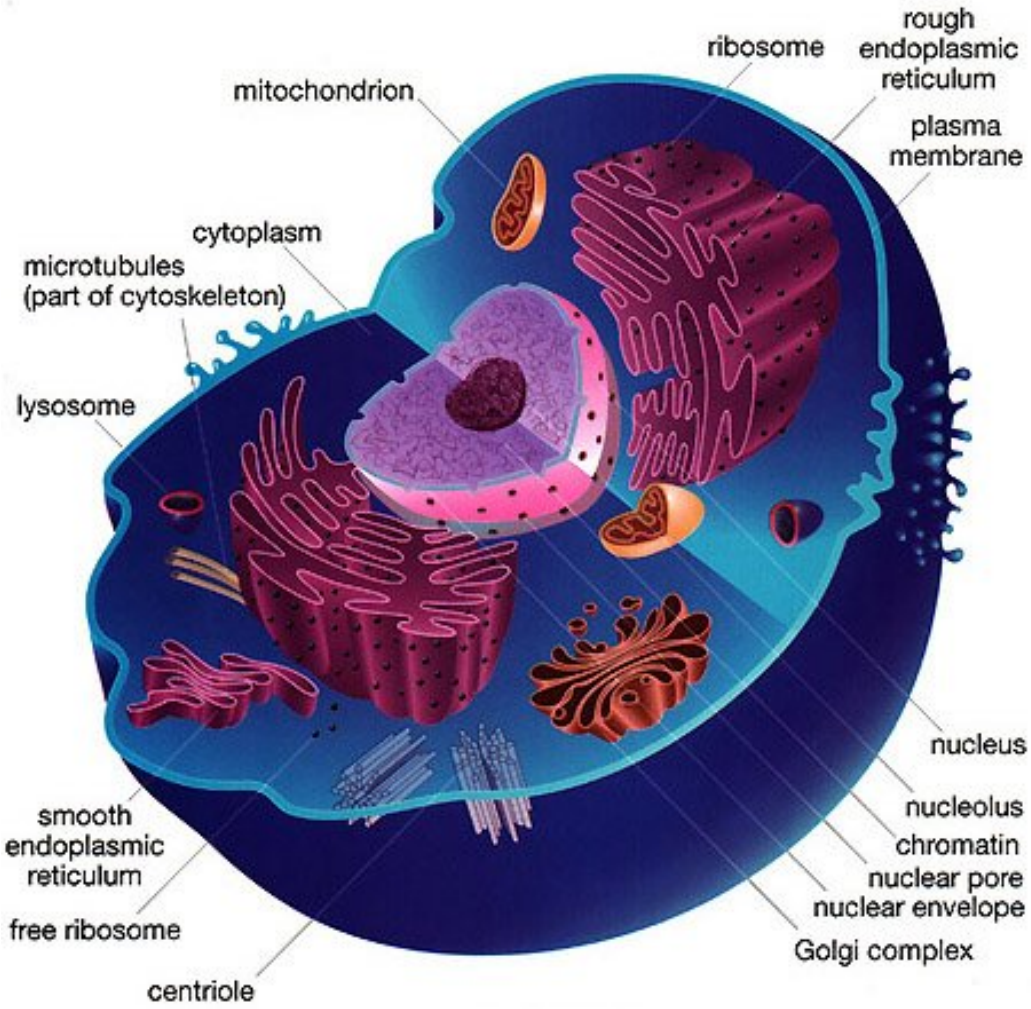
Our mission: distributed computing in biological cellular networks

Motivation

Selection of sensory organ precurser (SOP) cells = solving MIS
[Afek, Alon, Barad, Hornstein, Barkai, Bar-Joseph 11]



The structure of cells (eukaryotes)



The nucleus

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Analogous to central processing unit

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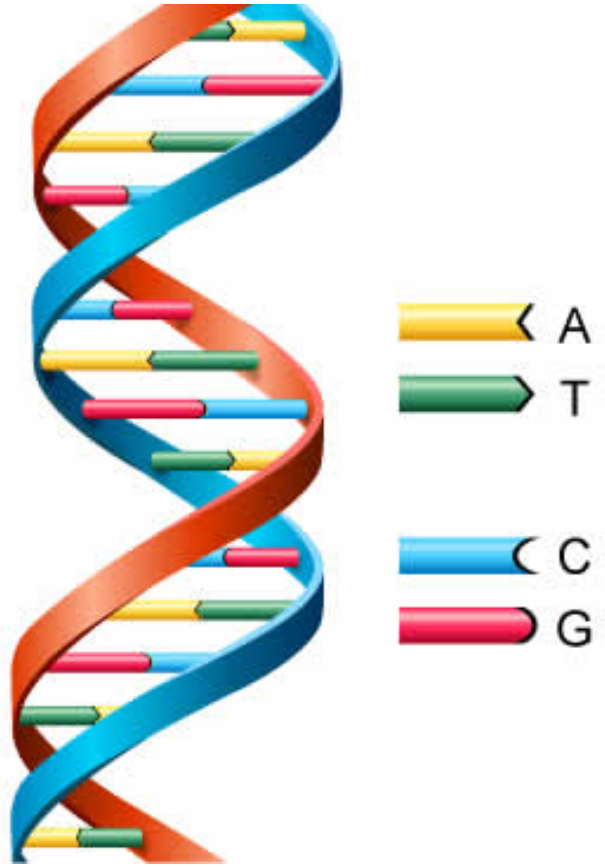

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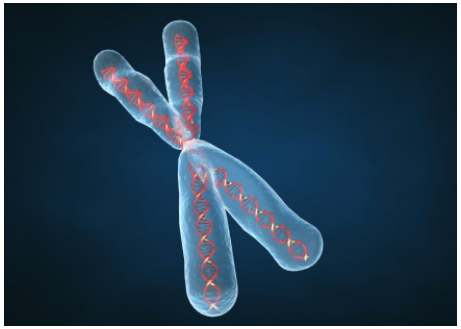

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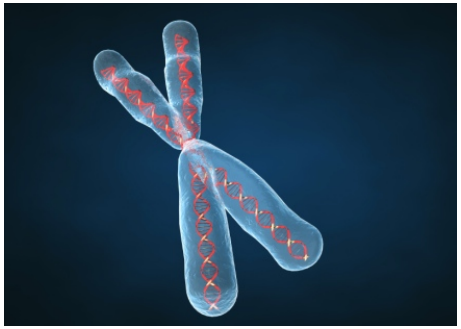

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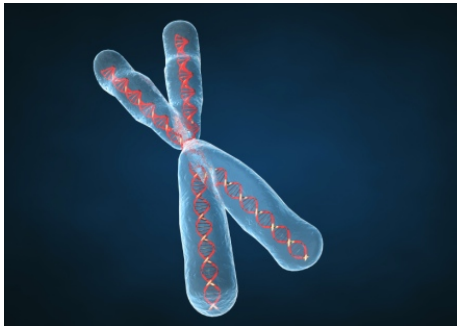

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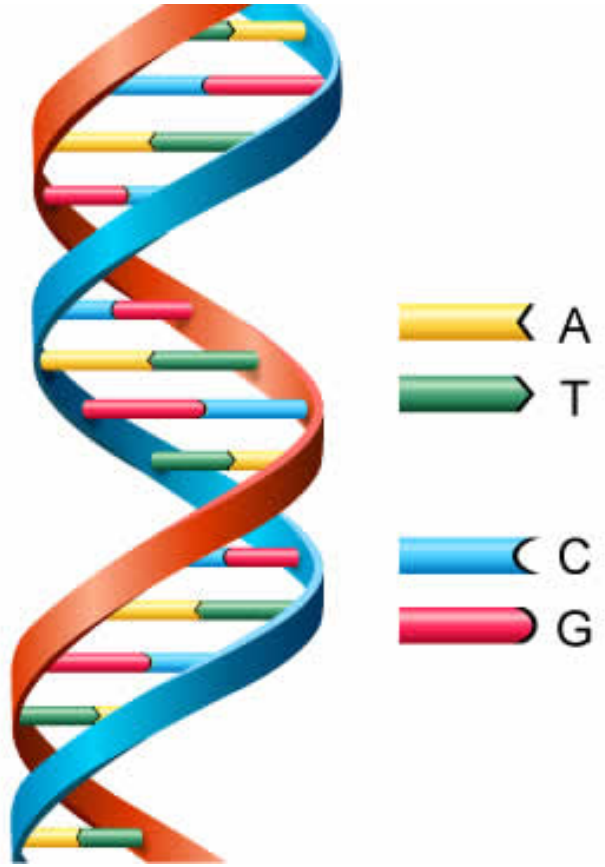

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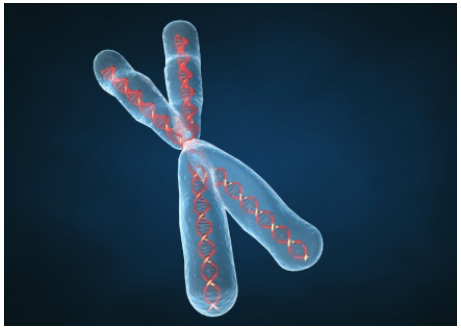

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- Execution = gene expression
  - Producing RNA molecules

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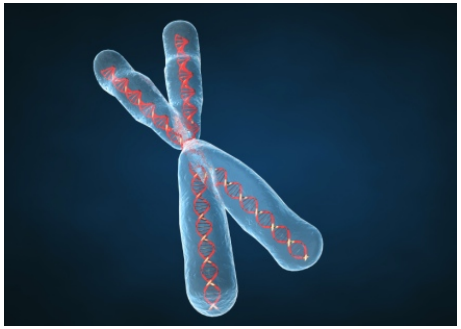

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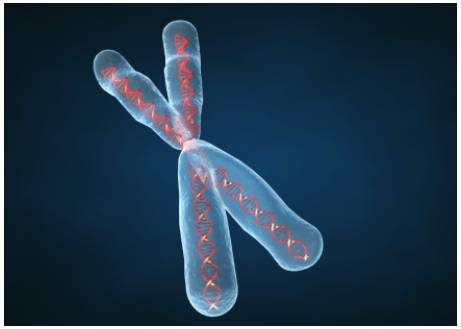

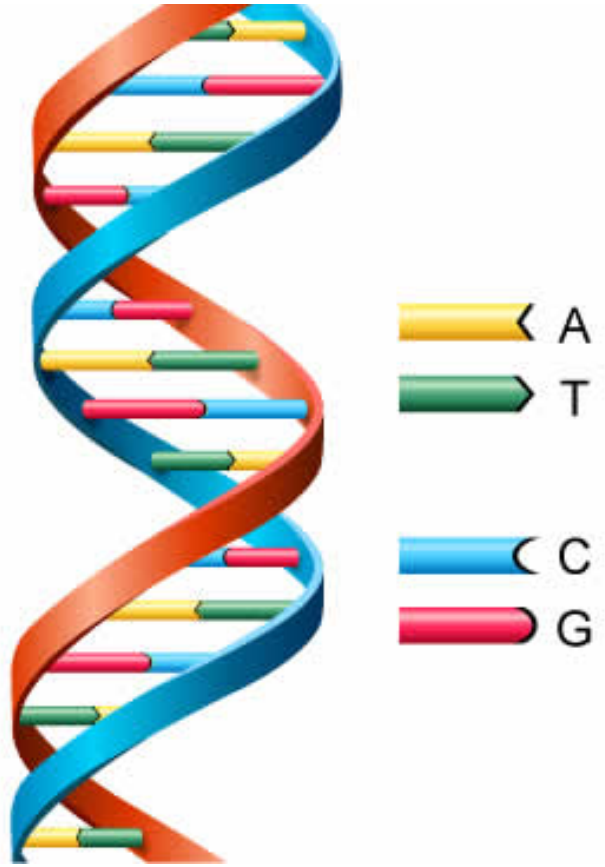

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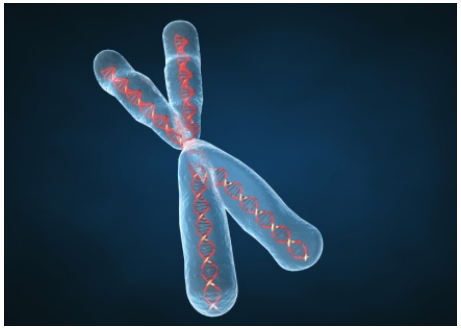

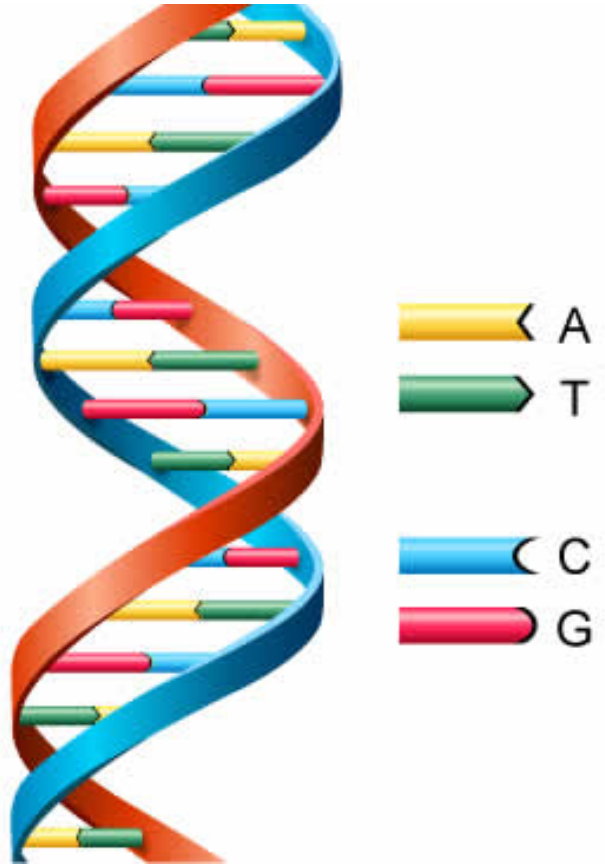

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  - Controlled by concentration levels

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  - respects network's topology

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Classify according to communication range:

- Juxtacrine — direct contact
  - respects network's topology
- Paracrine, endocrine
  - out of our scope

Juxtacrine communication

Delivery of message m from cell s to cell t

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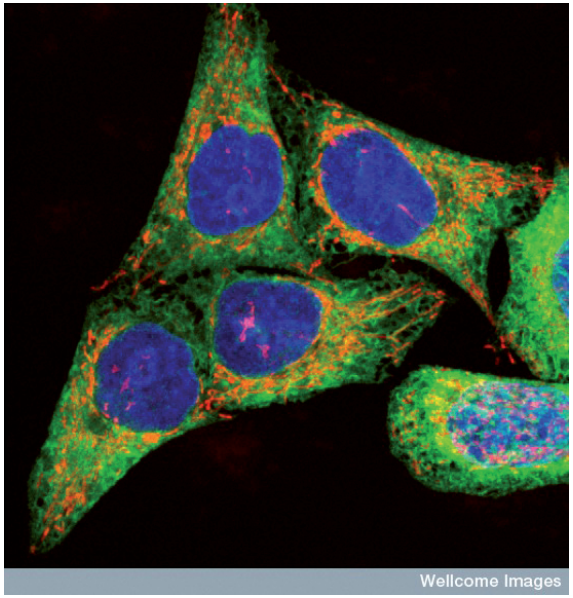

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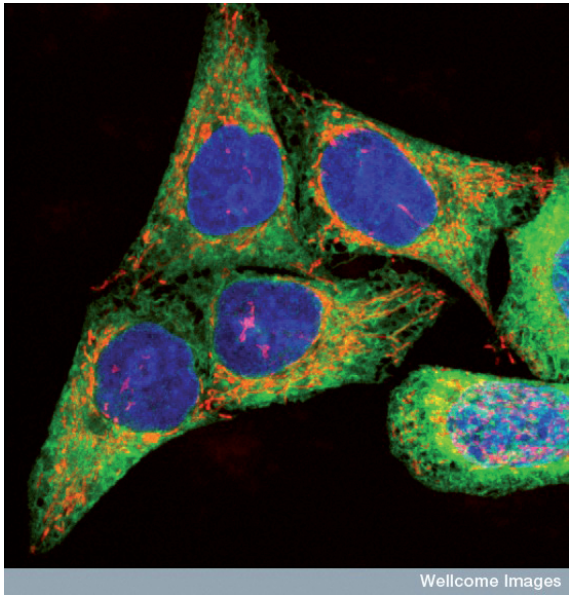

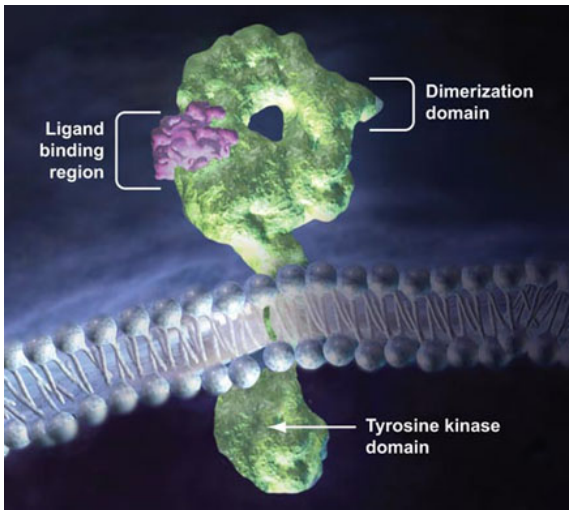

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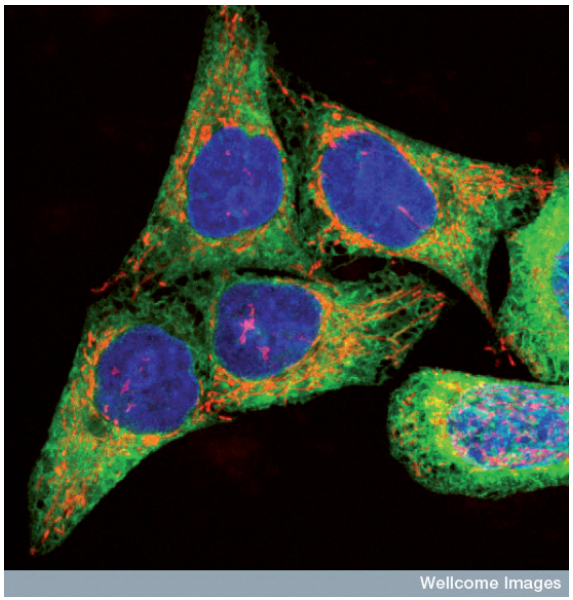

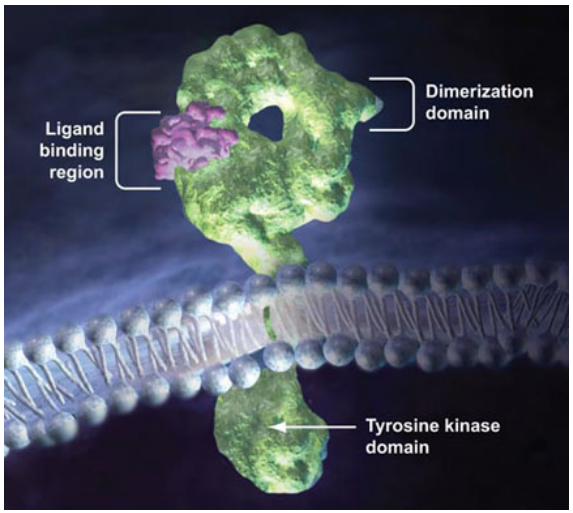

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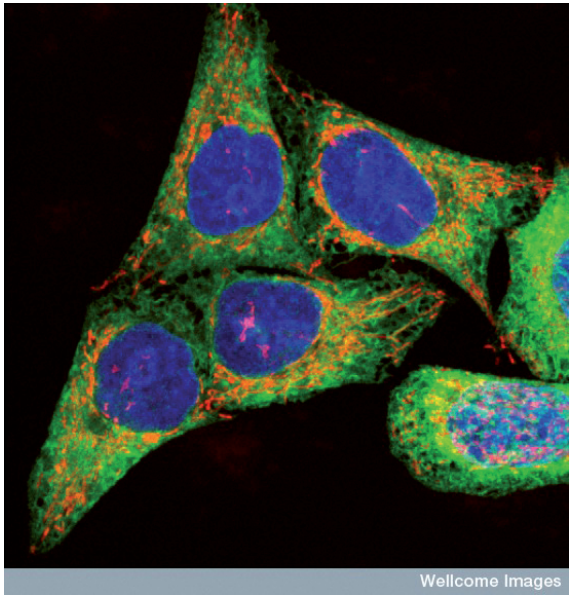

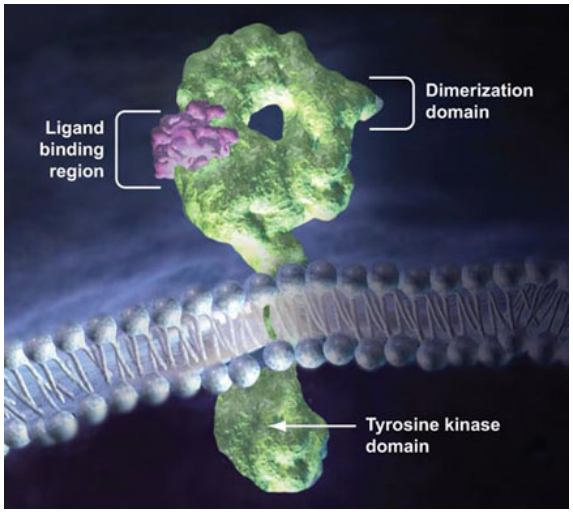

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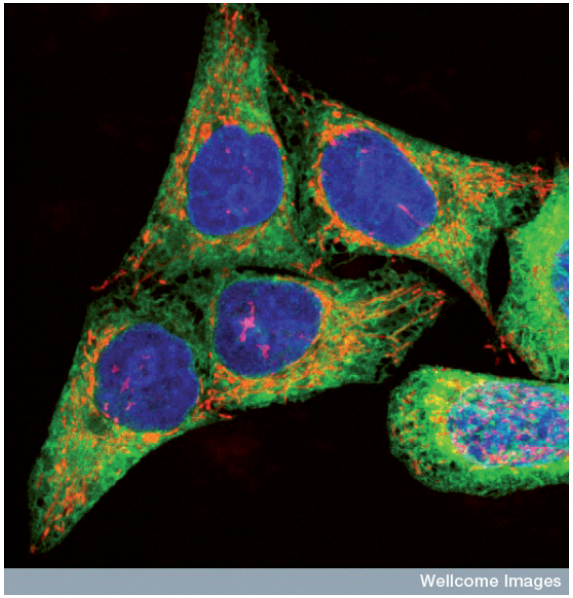

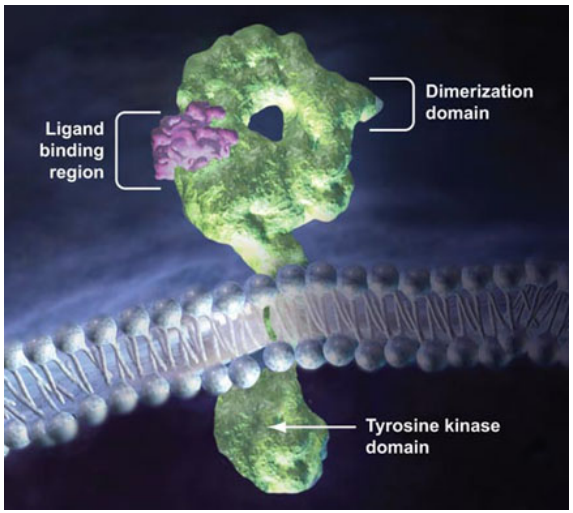

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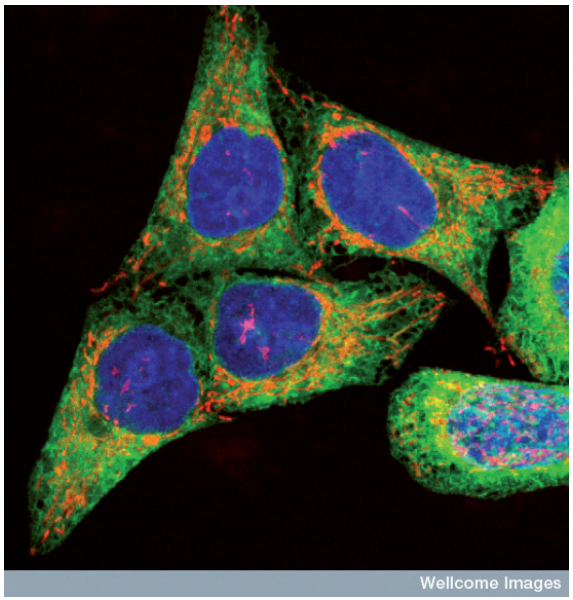
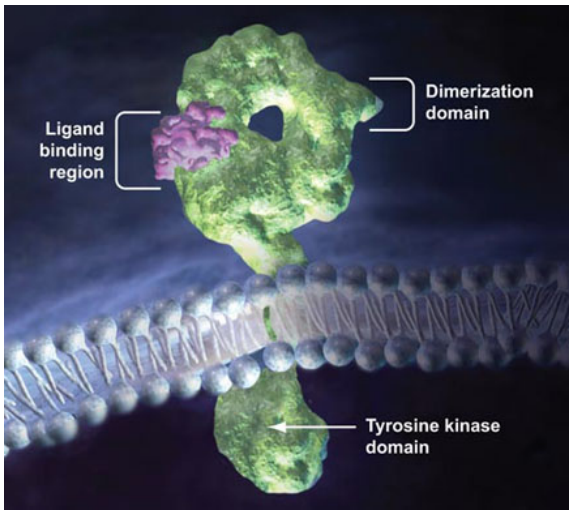


- Gap junction/receptor = port



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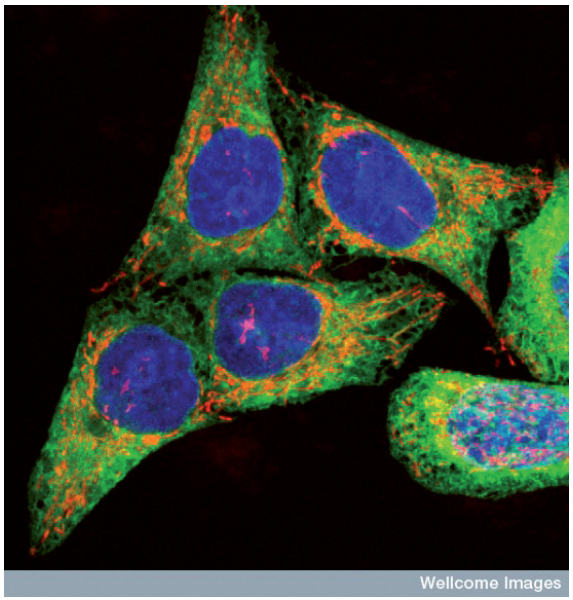

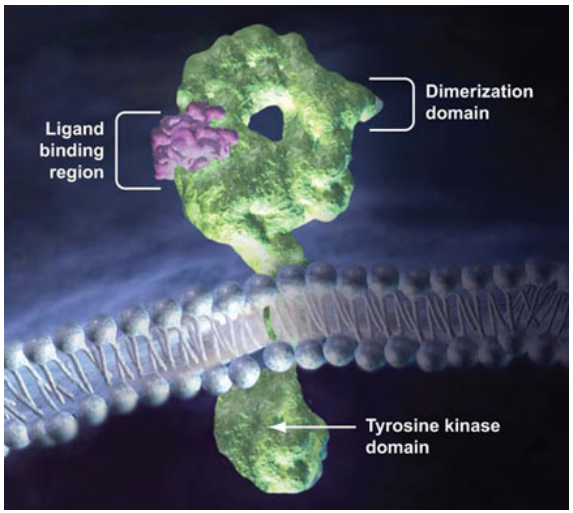

- Gap junction/receptor = port
- No sense of direction
  - all neighbors look the same

Message passing

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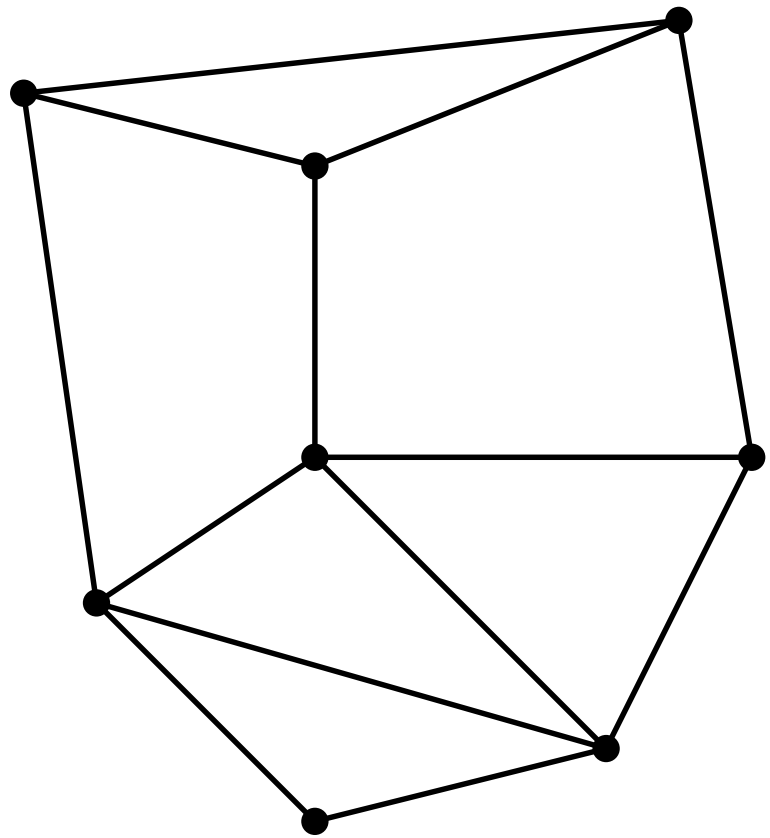
Nodes act locally (don't know global topology)



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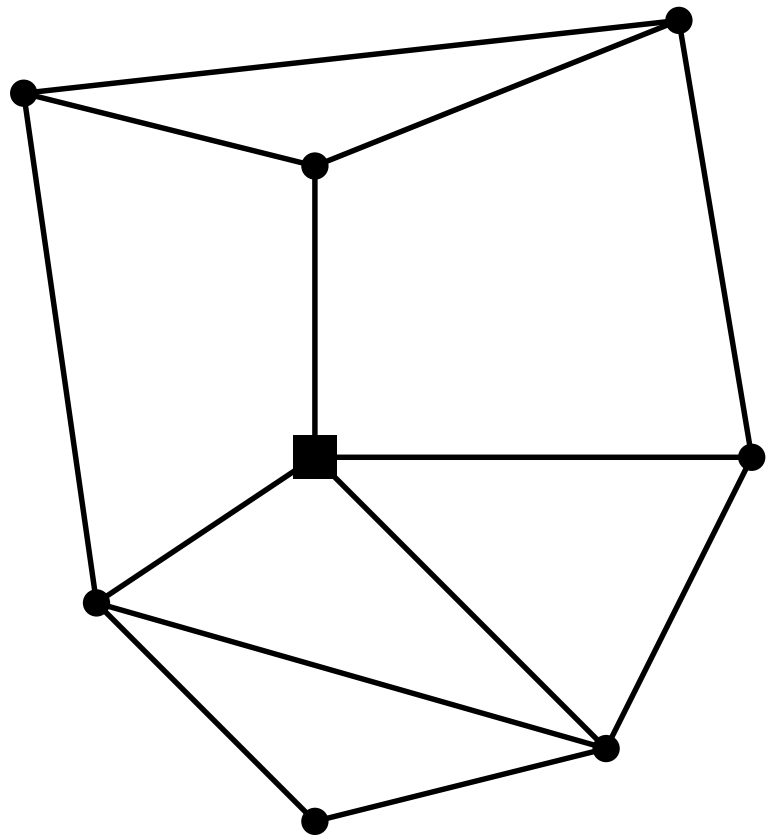

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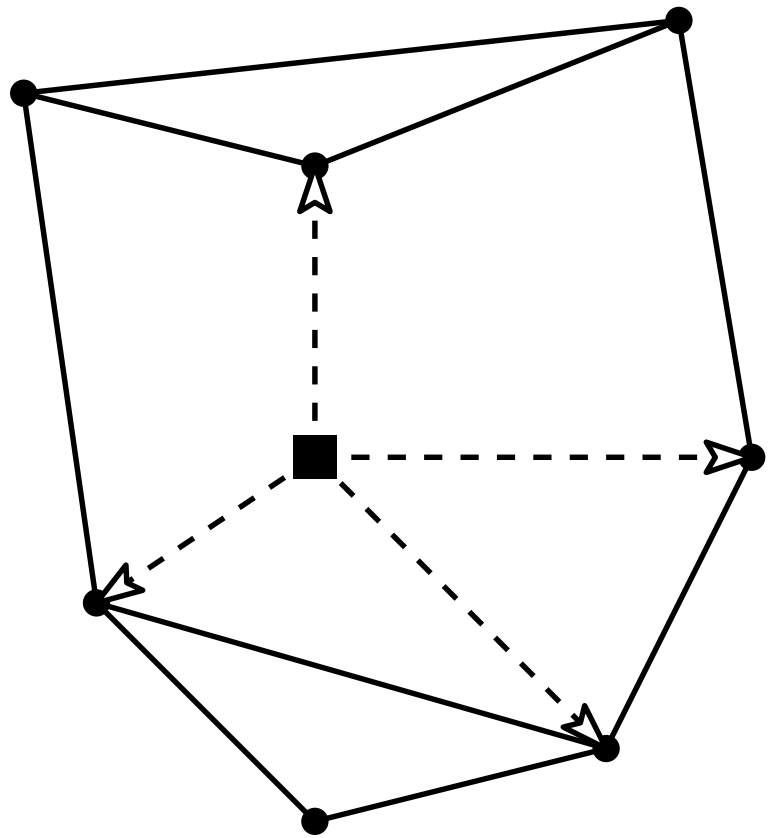

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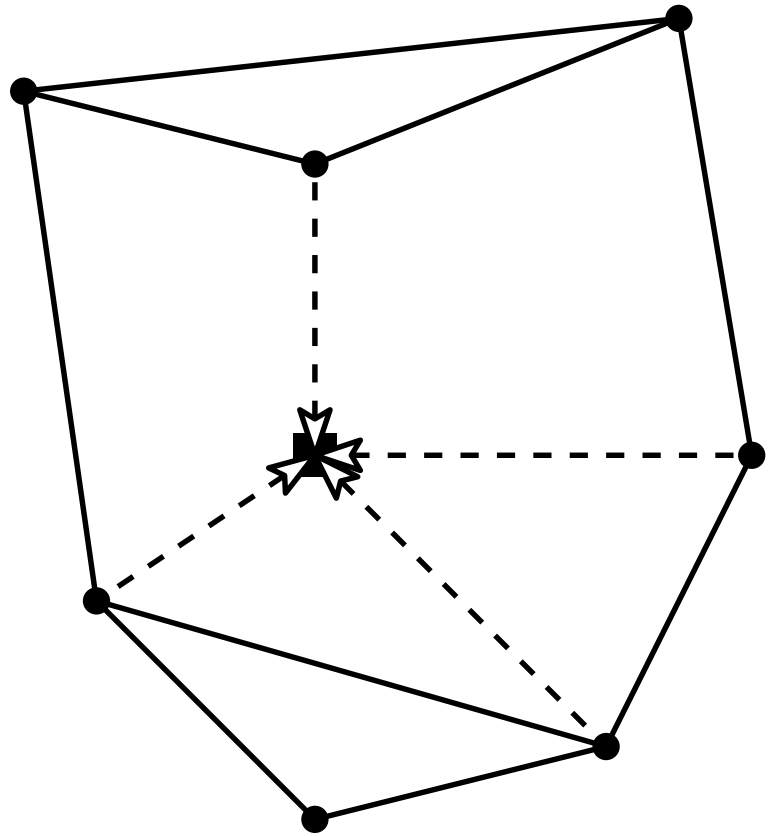

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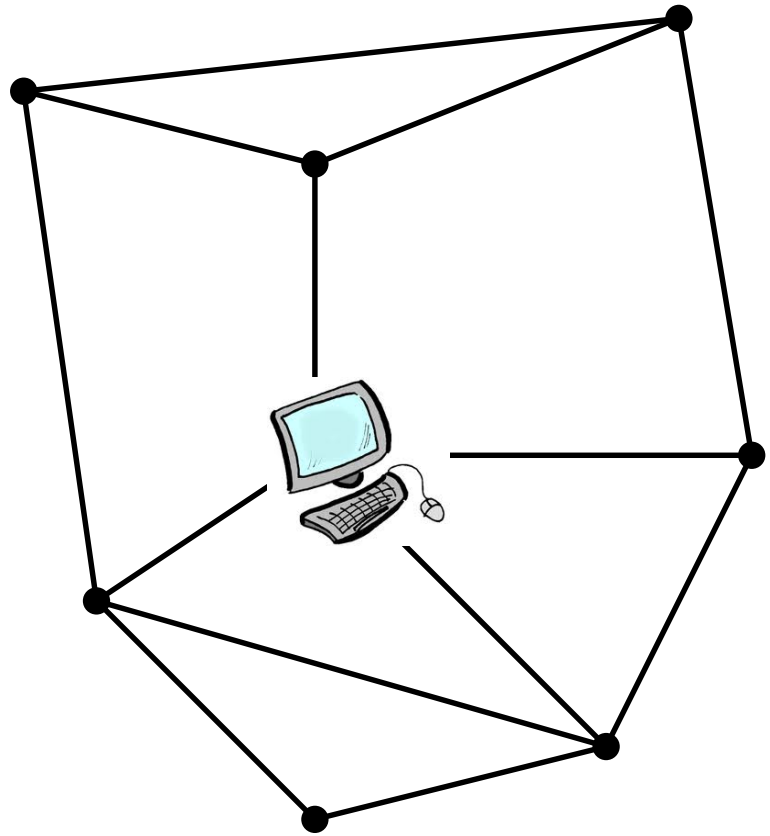

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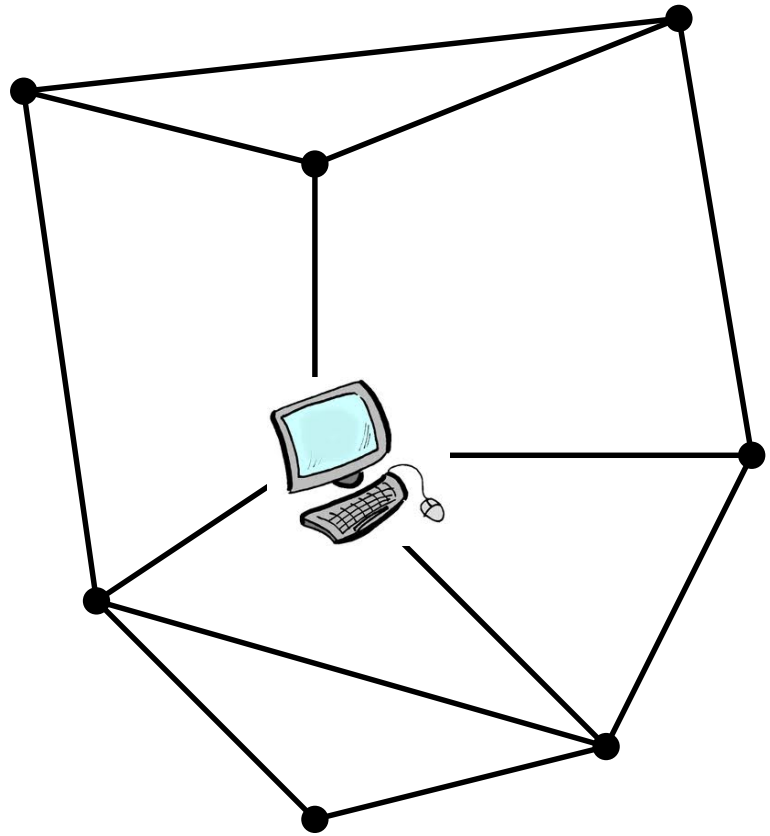

Communication too strong

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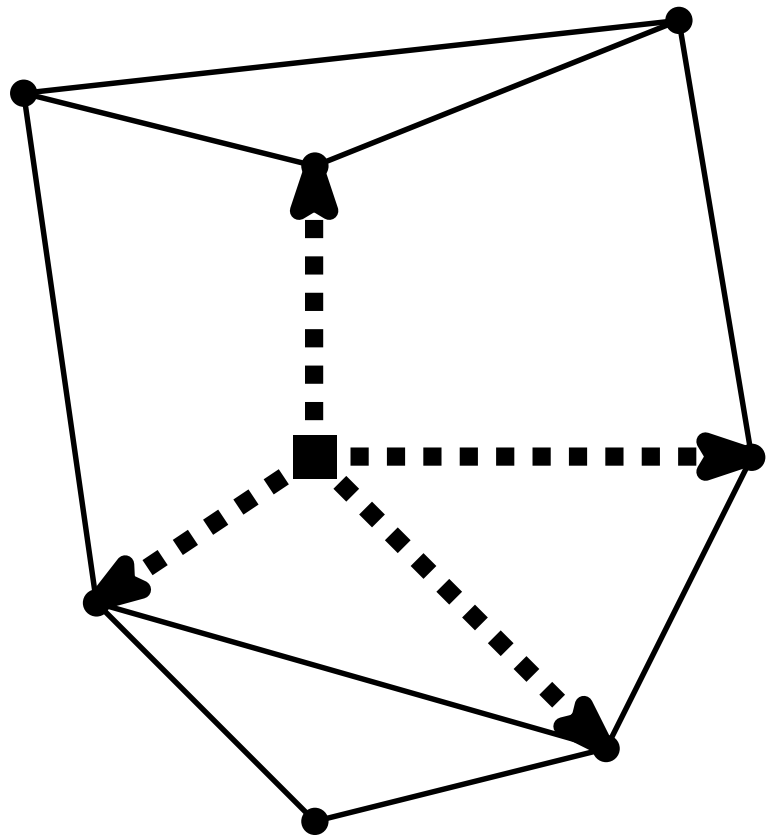

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- large messages (size grows with n)

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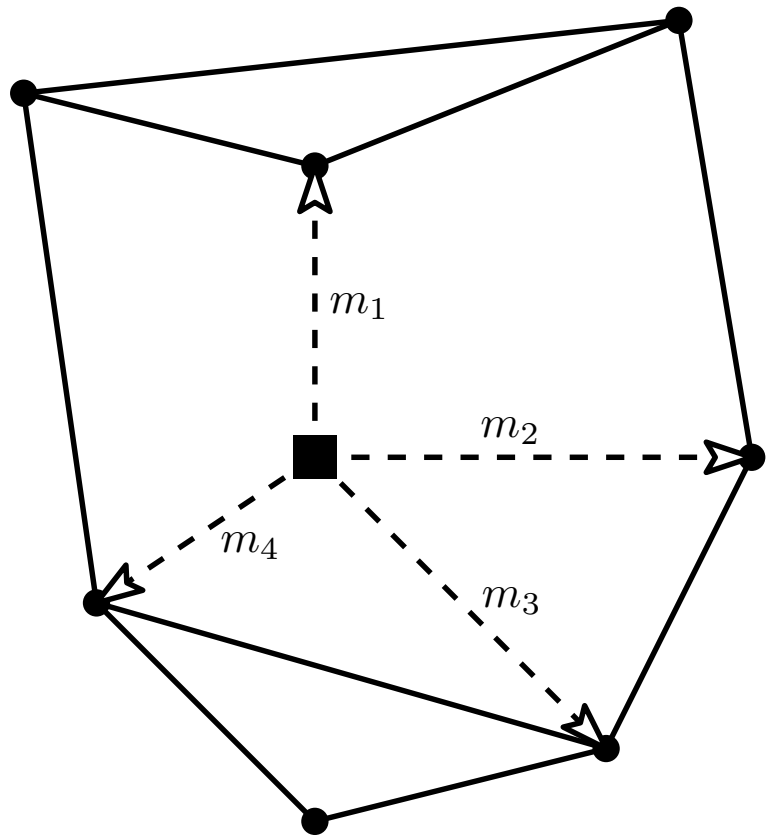

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- large messages (size grows with n)
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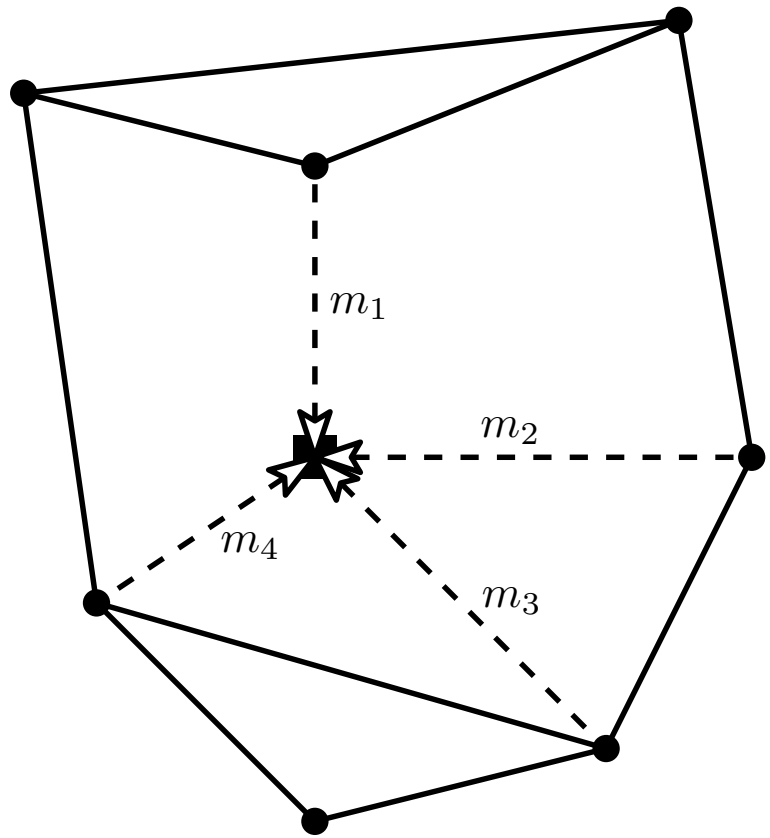

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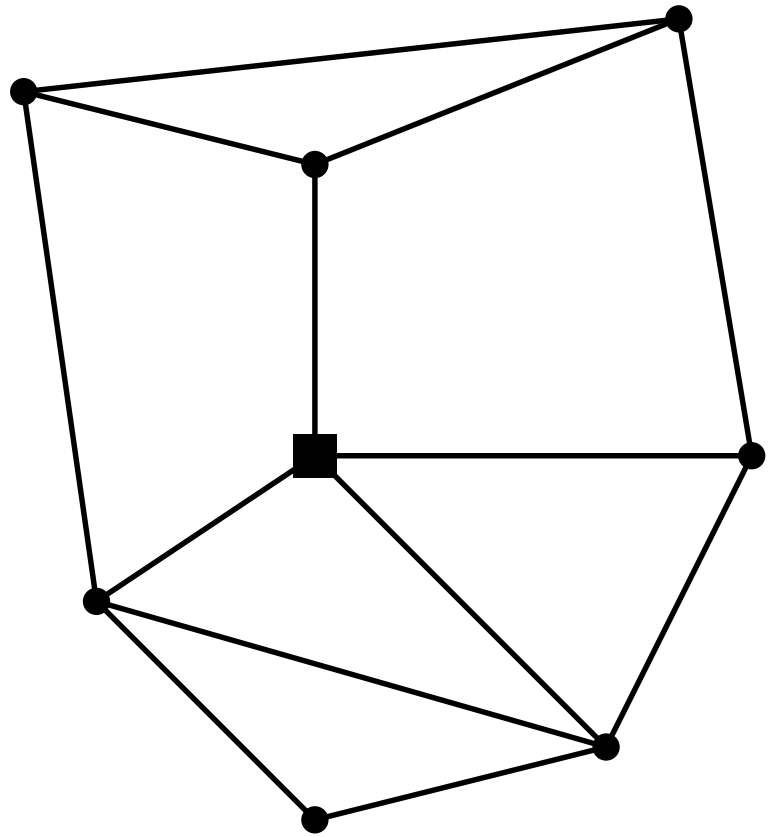
Introduced in [Cornejo, Kuhn 10]



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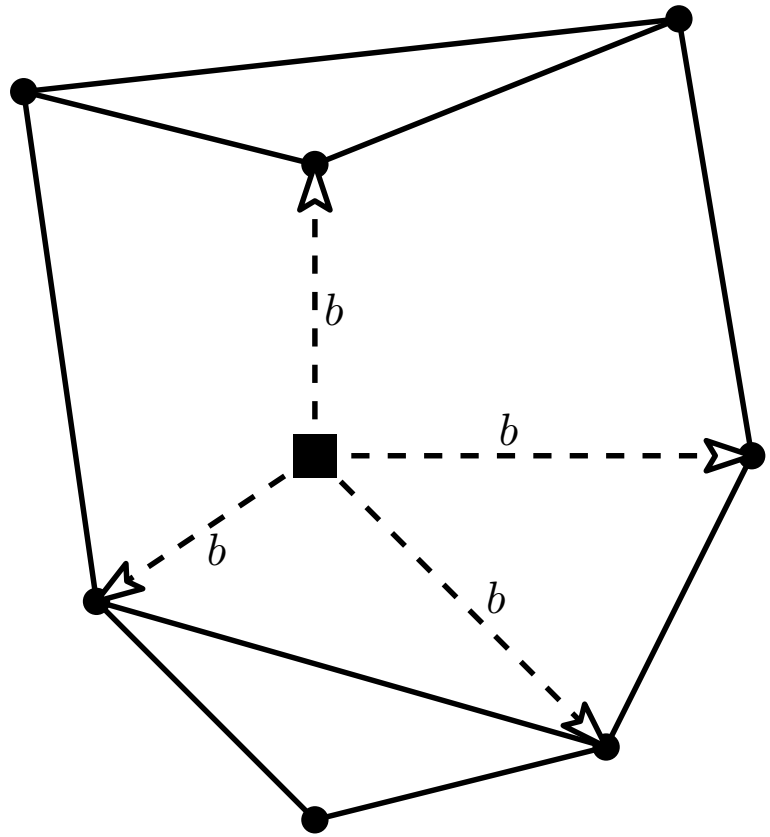

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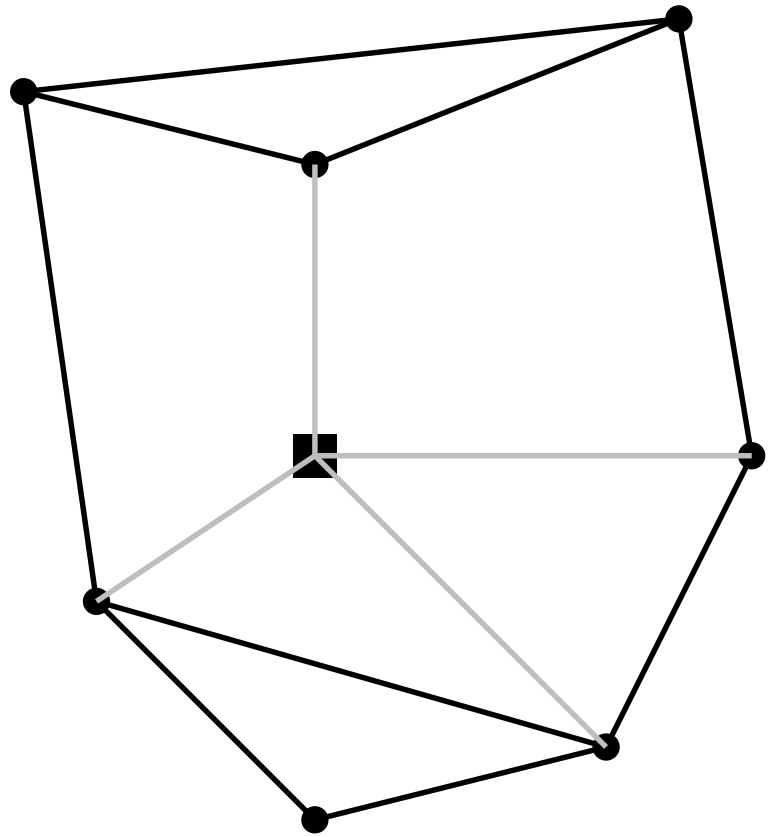

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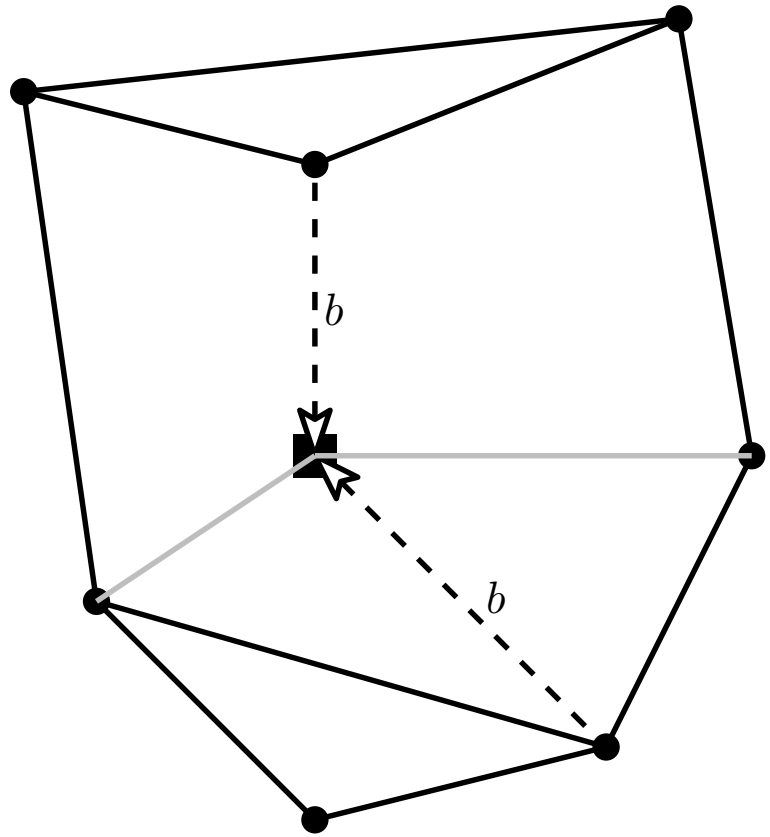

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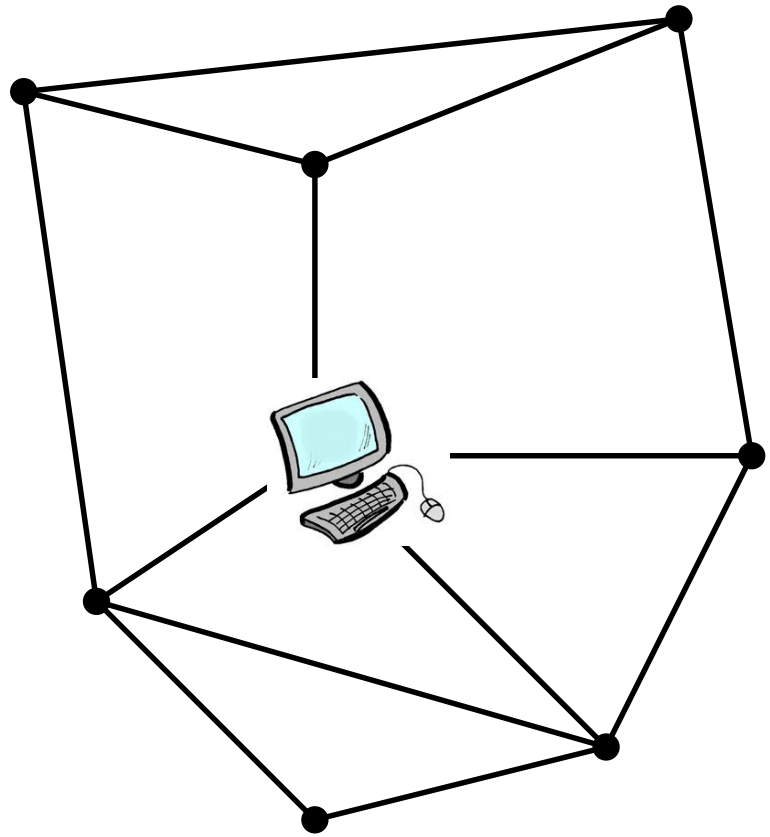

Local computation too strong

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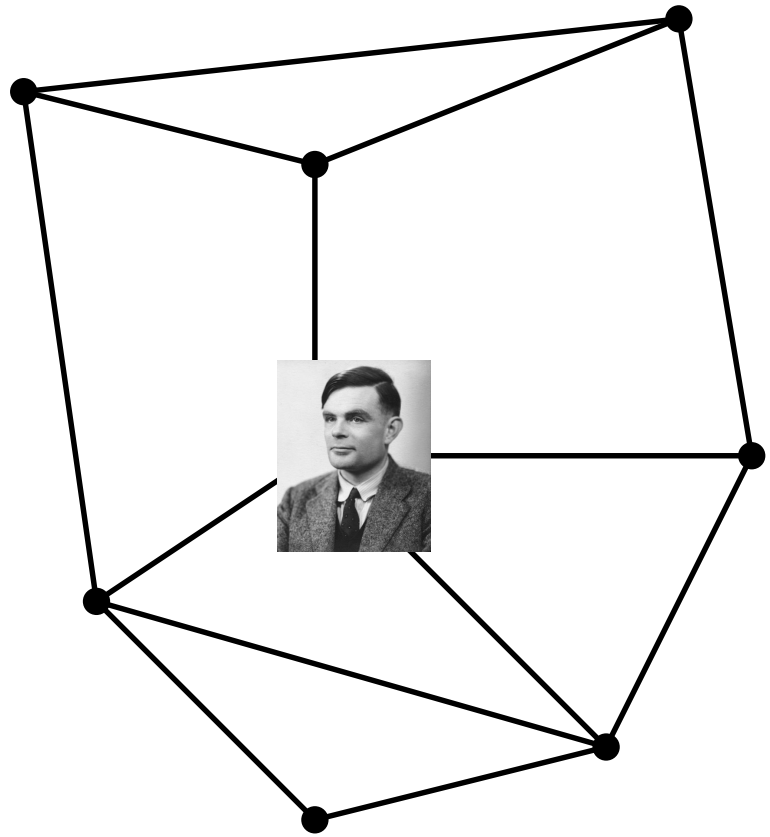

Local computation too strong

The beeping model

Introduced in [Cornejo, Kuhn 10]

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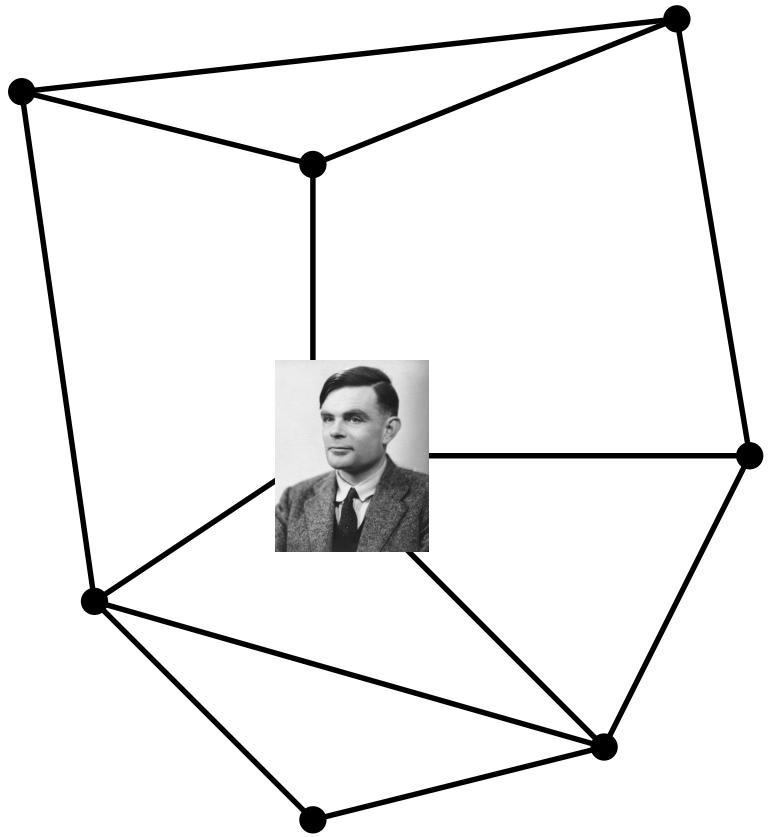

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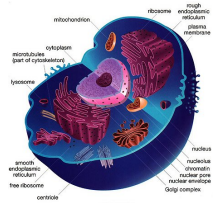

$\ll$

Finite state machines (a.k.a. automata)

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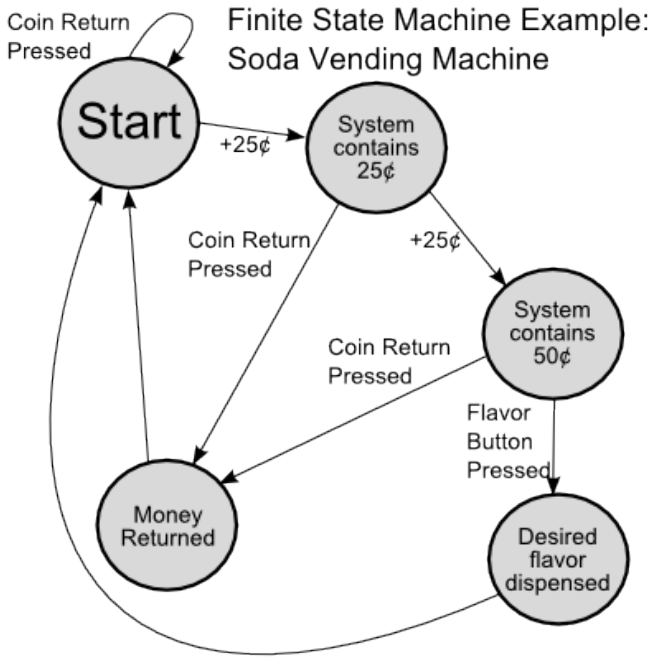

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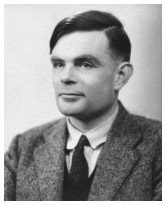

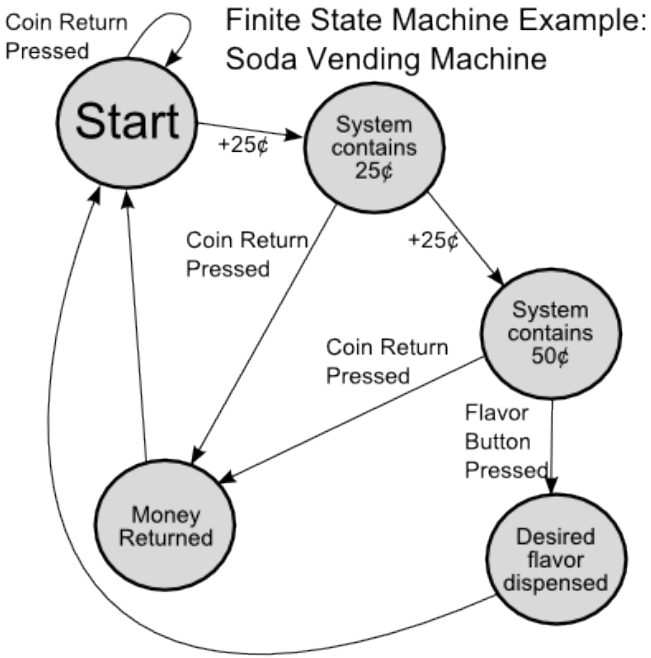

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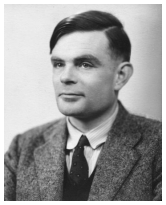

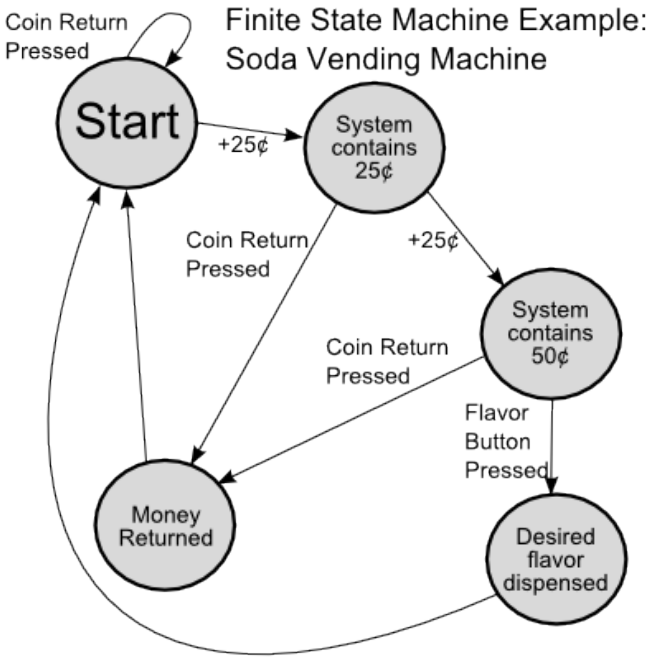

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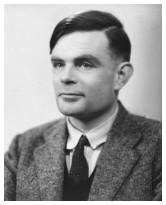

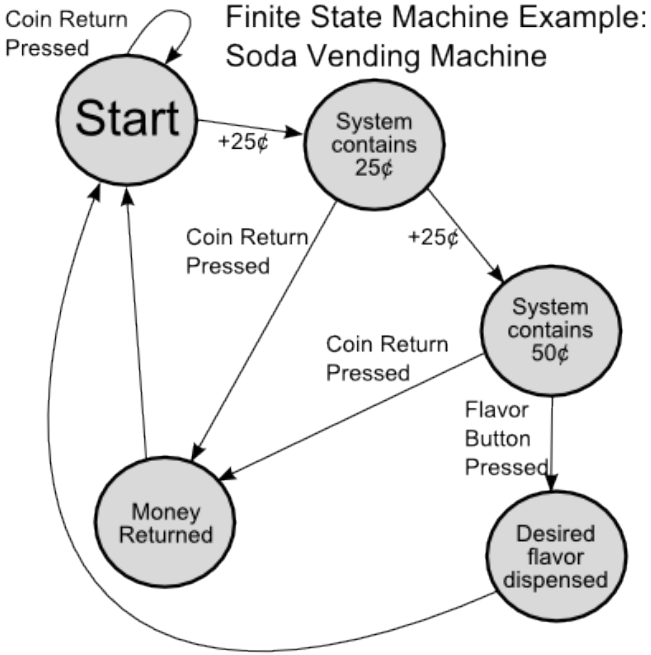

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Perhaps we should aim for a network of FSMs?

Cellular automata

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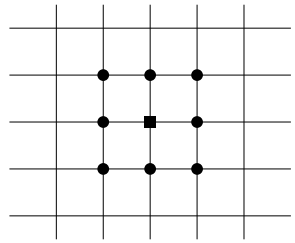

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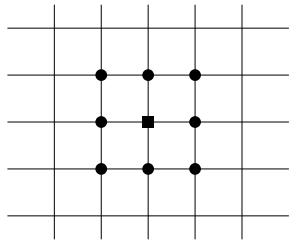

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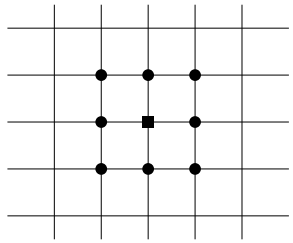

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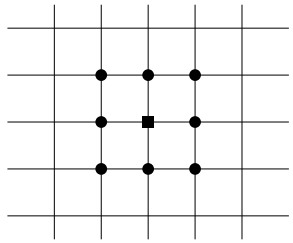

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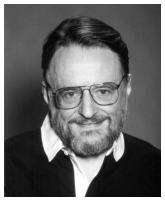

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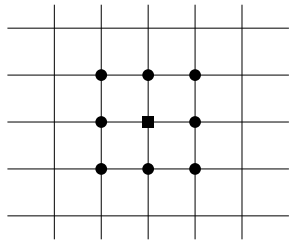

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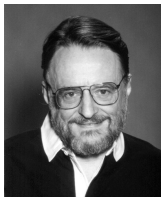
Game of life 

Digital physics  

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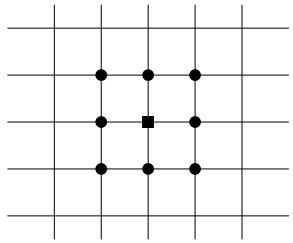

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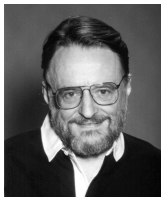

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 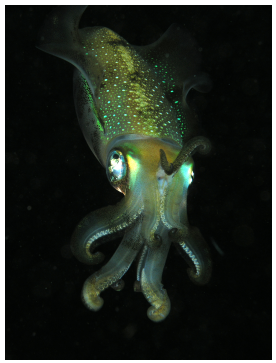

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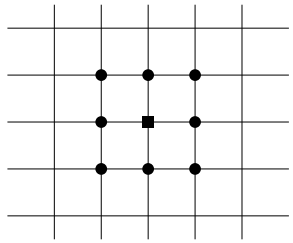

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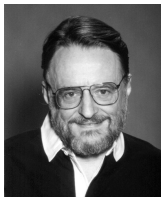
Game of life 

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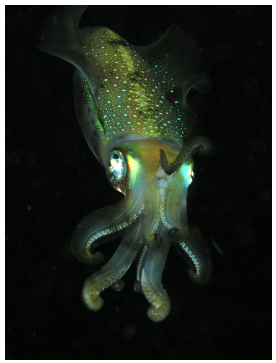
Biological processes  

Highly regular topology

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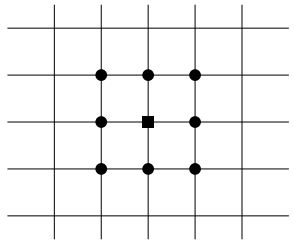

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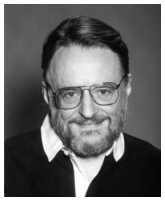
Game of life 

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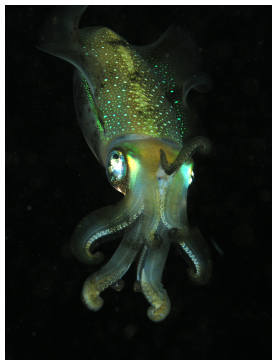
Biological processes  

Highly regular topology
Synchronous environment

1. Cell biology — a short intro
2. Abstract models
3. Networked finite state machines
   - Results
   - MIS protocol
4. Conclusions

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Corollary (Formal languages)

nFSM protocols do not exceed level 3 (out of 4) in Chomsky's hierarchy.

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The execution of a (randomized) linear-space Turing machine can be simulated by an nFSM protocol on a path.

Corollary (Formal languages)

nFSM protocols do not exceed level 3 (out of 4) in Chomsky's hierarchy.

Observation (Anonymous networks)

Leader election and consensus are impossible under nFSM.

Performance measure

Run-time:

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- # time units until all nodes terminate
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- Efficient algorithm $= \log^{O(1)} n$ run-time [Linial 92]

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Message passing model:

- $O(\log n)$ [Luby 86], [Alon, Babai, Itai 86]
- $\Omega\left(\sqrt{\log n}\right)$ [Kuhn, Moscibroda, Wattenhofer 04]
- Anonymous networks, constant size messages:
  $\Omega(\log n)$ [Kothapalli, Onus, Scheideler, Schindelhauer 06]
  $O(\log n)$ [Métivier, Robson, Sehab-Djahromi, Zemmari 11]

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The beeping model:

- $O(\log^2 n)$–$O(\log^3 n)$
  [Afek, Alon, Barad, Hornstein, Barkai, Bar-Joseph 11],
  [Afek, Alon, Bar-Joseph, Cornejo, Haeupler, Kuhn 11]

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Message passing model:

- $O(\Delta + \log^* n)$ [Barenboim, Elkin 09], [Kuhn 09]
- $\Omega(\log^* n)$ [Linial 92]
- $O(\log \Delta + \sqrt{\log n})$ [Schneider, Wattenhofer 10]
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- Anonymous (undirected) trees, constant size messages: $\Omega(\log n)$ [Kothapalli, Onus, Scheideler, Schindelhauer 06]
- Directed trees:
  - $O(\log^* n)$ [Cole, Vishkin 86]
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Simulating executions on the square graph

Lemma

For every nFSM protocol Π with bounding parameter $b = 1$, there exists an nFSM protocol Π^2 such that for every graph G, the execution of Π^2 on G simulates the execution of Π on G^2 with a constant multiplicative run-time overhead.

Efficient algorithms: 2-hop variants

Corollary (Maximal 2-hop independent set)

There exists an nFSM protocol that computes a maximal 2-hop independent set for any n-node graph in time $O(\log^2 n)$.

Corollary (2-hop coloring)

Given some constant d*, there exists an nFSM protocol that computes a 2-hop coloring with* (d^2+1) *colors for any n-node graph satisfying* $\Delta \leq d$ *in time* $O(\log n)$.

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Theorem

Maximal k-hop independent set and k-hop coloring are impossible in anonymous networks for any $k \geq 3$.

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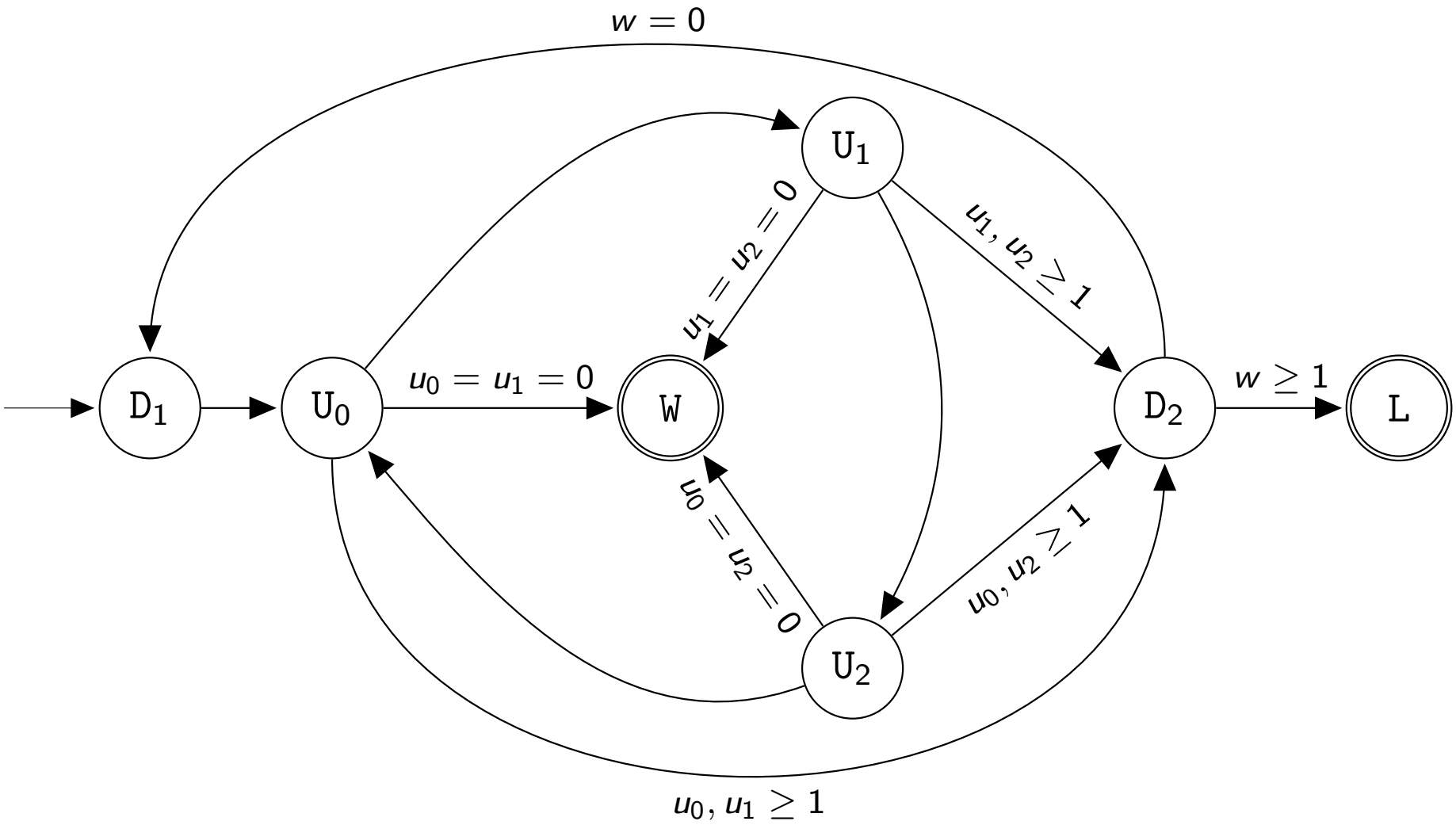
- How can we decide if u joins MIS without long aligned phases?

MIS under nFSM — solution



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  2. Guarantee fair competition $\Longrightarrow$
     const fraction of the edges is removed with const probability $\Longrightarrow$
     $O(\log n)$ tournaments w.h.p.

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